

AD-A187 533 PROBABILISTIC AND RELIABILITY DESIGN PROCEDURES FOR  
FLEXIBLE AIRFIELD PAV. (U) ARMY ENGINEER WATERWAYS  
EXPERIMENT STATION VICKSBURG MS DEOTE Y T CHOU

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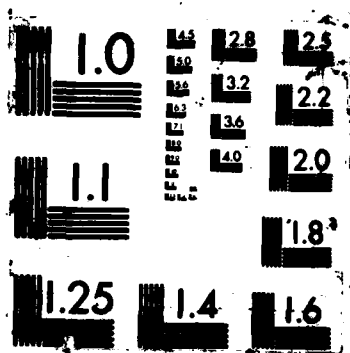
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# PROBABILISTIC AND RELIABILITY DESIGN PROCEDURES FOR FLEXIBLE AIRFIELD PAVEMENTS—ELASTIC LAYERED METHOD

by

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<p>A procedure was developed to analyze layered elastic flexible pavement systems in terms of probability and reliability. A computer program RELIBISA was prepared to carry out the computations. Rosenbluth method, instead of the conventional Taylor series expansion, is used to estimate the expected value and variance of the strains (dependent parameters) based on the input mean values of independent parameters, i.e., aircraft load, layer thicknesses, and material moduli. The relationships between the reliability level and the allowable strain repetition of the designed system which is established with results computed using RELIBISA provide a decision-making tool for engineers to design pavements at desired reliability level. The design can be optimized by selecting thicknesses of the bituminous concrete and the base layers so that the pavement is failed</p> <p style="text-align: right;">(Continued)</p>					
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in fatigue cracking and subgrade failure at nearly the same traffic level and the same reliability level. The reliability-strain repetition curves have steeper slopes with the bituminous concrete strain failure criterion than with the subgrade strain failure criterion, indicating that for flexible pavements designed using the Corps of Engineer's failure criteria, the design has a greater degree of uncertainty in preventing subgrade failure than fatigue cracking of the bituminous concrete surface course. However, this may not be true in real cases because the bituminous concrete failure criteria are determined based on controlled laboratory test data which do not consider the uncertainties existing in the laboratory-to-field correlations. The actual performance of the pavement with respect to fatigue cracking will be more uncertain than is considered in the design. The significance of the effect of failure criteria employed in the analysis on the derived conclusions is discussed and illustrated.

It was found that the performance of an all-bituminous concrete (ABC) pavement is sensitive, in the descending order, to variations of the input parameters ABC thickness  $h_1$ , gear load  $P$ , ABC modulus  $E_1$ , and subgrade modulus  $E_2$  for both bituminous concrete and subgrade strain criteria. The performance of a conventional flexible pavement is sensitive, in the descending order, to variations of gear load  $P$ , the thickness of the granular base  $h_2$ , the subgrade modulus  $E_3$ , the thickness and modulus of the bituminous concrete surface course  $h_1$ , and  $E_1$ , respectively, for the subgrade strain failure criterion, and to variations of  $P$ ,  $h_1$ ,  $E_1$ ,  $E_2$ ,  $h_2$ , and  $E_3$  for the bituminous concrete strains failure criteria. Although the pavement performance is more sensitive to the variation of layer thickness than to that of the elastic modulus of the layer, the actual variation of material moduli in the field is known to be much larger than the variation of layer thickness. Strict control during construction is recommended to reduce the degree of material variabilities and thus to lessen the degree of uncertainty and to increase the confidence level of the designed pavement.

## PREFACE

The work reported herein was funded by the Office, Chief of Engineers, US Army, under the FY 82 RDTE Program, Project No. 4A161102AT22, Task AO, Work Unit 009, "Methodology for Considering Material Variability in Pavement Design." Mr. S. L. Gillespie, US Army Corps of Engineers, was the Technical Monitor.

The study was conducted by the US Army Engineer Waterways Experiment Station (WES), Geotechnical Laboratory (GL) by Dr. Y. T. Chou, Pavement Systems Division (PSD). The work was under the general supervision of Dr. W. F. Marcuson III, Chief, GL, and Mr. H. H. Ulery, Jr., Chief, PSD. This report was written by Dr. Chou. Ms. Odell F. Allen, Information Products Division, Information Technology Laboratory, edited this report.

COL Allen F. Grum, USA, was the previous Director of WES. COL Dwayne G. Lee, CE, is the present Commander and Director. Dr. Robert W. Whalin is Technical Director.

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CONVERSION FACTORS, NON-SI TO SI (METRIC)  
UNITS OF MEASUREMENTS

Non-SI units of measurement used in this report can be converted to SI  
(metric) units as follows:

<u>Multiply</u>	<u>By</u>	<u>To Obtain</u>
feet	0.3048	metres
inches	2.54	centimetres
pounds (force)	4.448222	newtons
pounds (force) per square inch	6.894757	kilopascals
pounds (mass)	0.4535924	kilograms

PROBABILISTIC AND RELIABILITY DESIGN PROCEDURES  
FOR FLEXIBLE AIRFIELD PAVEMENTS--  
ELASTIC LAYERED METHOD

PART I: INTRODUCTION

Background

1. The design of flexible airfield pavements in the US Army Corps of Engineers (USACE) is currently based on two methods: (a) the California Bearing Ratio (CBR) equation that is empirical in nature and yields a design thickness for a given design condition, and (b) the multilayered elastic method that is analytical in nature and yields stresses, strains, and deflections in the pavement system for a particular loading condition and pavement geometry which in turn are compared with established failure criteria to determine the performance of the given pavement. Both design method approaches are deterministic, i.e., a unique pavement system is designed for the specific set of input variables necessary to solve the problem. The input variables are unique in their magnitudes. In the CBR method, a pavement thickness is determined from given values of subgrade CBR, gear load and configuration, tire contact area, and design coverage level. The effect of material variability on pavement performance is considered in the designer's selection of the subgrade CBR value, and the design safety factor is implicitly contained within construction specifications such as compaction requirements. However, a quantification of these effects can be accomplished using a probabilistic and reliability approach, and the design procedures can be improved by showing the partial effect of each design parameter.

Purpose

2. The purpose of this study is to develop a design methodology that has the capability of considering design parameter variability in the USACE design procedure for flexible airfield pavements using the elastic layered method. The design procedure is expressed in probabilistic and reliability terms, i.e., the design pavement thicknesses at different performance levels

are computed for a range of reliability levels. The designer can select the pavement thickness and in some cases develop an overlay design scheme based on the desired reliability level. The design procedure is incorporated in a computer program. By using the procedure, the partial effect of the variability of each design parameter on pavement performance can also be investigated, and its effects on the final design can be quantified. Emphasis can be placed on the crucial parameters to be tightly controlled in the construction phases and/or the crucial loading parameters dictated by the intended use of the pavement.

### Scope

3. The computer program RELIBISA is a modified version of the layered elastic program BISAR (Koninklijke/Shell Laboratorium 1972) for the application of the Rosenblueth (1975) method in pavement analysis. Current failure criteria (Barker and Brabston 1975) for bituminous concrete (tensile strains at the bottom of the surface layer) and subgrade soil (vertical strain at the top of the subgrade) used by the USACE are incorporated in the program. The program is capable of using other failure criteria and loadings other than aircraft. The computed results of the program are the allowable strain repetitions of the design pavement structure under the specific aircraft loads computed at different reliability levels under a given set of material properties and input parameter variabilities. The procedure treats the aircraft loads, pavement thicknesses, and material properties as stochastic variables, and the variables are input by their means and coefficients of variation. Two example pavements, one an all-bituminous concrete pavement and the other a three-layer flexible pavement, were analyzed using RELIBISA program to illustrate the merit of pavement design in terms of probability and reliability. The relative effect of each design parameter variability on pavement performance was evaluated and discussed. The variabilities of input parameters and the significance of failure criteria employed in the analysis on designed pavements are discussed.

## PART II: PREVIOUS WORKS

4. In 1983, Witczak, Uzan, and Johnson (1983) of the University of Maryland, under a contract from the Waterways Experiment Station (WES), developed design methodologies for rigid airfield pavements in terms of probability and reliability. The work involved design procedures based on the Westergaard free edge stress slab theory (Westergaard 1948) and the multilayer elastic theory (Parker et al. 1979). Taylor series expansion (Benjamin and Cornell 1970) was used in the probabilistic analysis. To simplify the complex procedure that is inherent in the Taylor series expansion and the multiple subbase layer, two major investigations (or simplifications) were involved: (a) the use of the composite modulus of subgrade reaction to expand the procedure to solve problems having multiple subbase layers and (b) the evaluation of a general equation form derived from the multiple regression analysis of numerous computations used to predict maximum stresses in the concrete slab for various aircraft types. It was found that the composite modulus depends not only upon the design parameters of the layered (subbase/subgrade) system underneath the rigid pavement, but also upon the pavement thickness (for constant elastic properties of the concrete) and the loading conditions (number of wheels in the gear). The regression equations of the maximum tensile stress at the bottom of the concrete layer were found highly accurate, allowing their use without any correction for both design purposes and probabilistic/reliability analyses. Computer programs were developed including the composite modulus, regression equations, and Taylor series expansion and can be used as follows: (a) in the analysis in probabilistic/reliability terms for a given pavement system and loading aircraft. The means and coefficients of variation of the design parameters serve as the input. The computer program produces values of the number of coverages and their reliability levels. (b) in the design of a rigid pavement for a given reliability. The number of coverages is also an input so that the slab thickness may be determined for the given reliability and coverage level.

5. The development of probabilistic and reliability design methodologies for flexible airfield pavements has been conducted at WES. Effort was first conducted on the CBR design method (Fergus 1950; Turnbull and Ahlvin 1957; Hammitt et al. 1971). The work is reported in a technical report (Chou 1986) which is described in the next paragraph.

6. The original and the new CBR equation for flexible airfield pavements are shown below as Equations 1 and 2, respectively.

$$t = \alpha \sqrt{\frac{P}{8.1 \text{ CBR}} - \frac{A}{\pi}} \quad (1)$$

$$t = \alpha \left\{ \sqrt{A} \left[ -0.0481 - 1.1562 \left( \log \frac{\text{CBR} \cdot A}{P} \right) - 0.6414 \left( \log \frac{\text{CBR} \cdot A}{P} \right)^2 - 0.473 \left( \log \frac{\text{CBR} \cdot A}{P} \right)^3 \right] \right\} \quad (2)$$

where

- $t$  = pavement thickness
- $\alpha$  = a traffic factor
- $P$  = single-wheel load (or the equivalent single-wheel load (ESWL) in the case of the multiple-wheel loads)
- CBR = California Bearing Ratio of the subgrade soil
- $A$  = tire contact area

Equation 1 was formulated in the 1950's, and Equation 2 is the new form formulated in the early 1970's based on additional test data (Hammitt et al. 1971). The design parameters considered were the load  $P$  (or the ESWL), the subgrade CBR, the tire contact area  $A$ , and the pavement total thickness  $h$ . The expected value and the variance of the dependent variable traffic factor  $\alpha$  were estimated using the Taylor series expansion and the Rosenblueth method. Differences in computed results between the two methods were found to be small, although the derivation of the expressions for Taylor series expansion is very complicated. A computer program was developed to estimate the reliability of the designed pavement system based on known variabilities of design parameters. Results of the reliability analysis indicate that prediction of pavement performance is most influenced by variations of pavement thickness and is least influenced by variations of tire contact area  $A$ . The effects of variations of wheel load  $P$  and subgrade CBR are identical. The weighting factors for parameters  $t$ , CBR,  $P$ , and  $A$ , in general cases, are approximately 1, 0.34, 0.34, and 0.01, respectively. It was thus concluded that in the future analysis of pavements involving input parameter variabilities, the

effect of the variation of wheel contact area may be neglected. It was also recommended that strict quality control be exercised during construction to reduce variations of pavement thickness and subgrade CBR, and that the Rosenblueth method (Rosenblueth 1975) be used because of its simplicity and accuracy in the probabilistic analysis of layered elastic system.

7. Related publications in pavement structures in terms of probability and reliability have been written by George and Nair 1982; Darter and Hudson, (1973); Hudson (1975); Kennedy, Hudson, and McCullough (1975); Treybig, Hudson, and McCullough (1970); Johnson (1983); Darter, Hudson, and Hass (1974);\* Holsen and Hudson (1974); Hutchinson (1966); Hutchinson and Hass (1968); Jain, McCullough, and Hudson (1971); Kher and Darter (1973); Kher, Hudson, and McCullough (1970, 1971); Lytton and McFarland (1974); Lytton, McFarland, and Schafer (1975); Schrivner et al. (1968); Potter (1985); and Vanmarcke (1979).

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\* M. I. Darter, W. R. Hudson, and R. C. G. Hass. 1974 (Jan.) "Selection of Optimal Pavement Designs Considering Reliability, Performance, and Costs," Unpublished paper prepared for the 53rd Annual meeting of the Highway Research Board, Center for Highway Research, University of Texas, Austin, Tex.

### PART III: PROBABILISTIC AND RELIABILITY APPROACH

#### General

8. In analyzing a pavement structure in probabilistic and reliability terms, the expected value and variance of a function (such as the computed stresses, strain, or load repetition) should first be determined, and the reliability of the design can then be evaluated. The Taylor series expansion (Benjamin and Cornell 1970) and the Rosenblueth (1975) procedure are generally used. These methods are presented below.

#### Taylor Series Expansion

9. The Taylor formula for the expansion of a function  $f(x)$ , which has  $N$  continuous derivatives, about the function's mean  $\mu$  is

$$f(x) = f(\mu) + f'(\mu)(x - \mu) + \frac{f''(\mu)}{2} (x - \mu)^2 + \dots \text{higher order terms} + \text{remainder} \quad (3)$$

Since the expected value of  $(x - \mu)$  is zero and the expected value of  $(x - \mu)^2$  is the variance\* of  $x$ , i.e.,  $E(x - \mu) = 0$  and  $E(x - \mu)^2 = \sigma_x^2$ , the expected value of  $f(x)$  becomes

$$E[f(x)] = f(\mu) + 0 + \frac{1}{2} f''(\mu) \sigma_x^2 + \dots$$
$$E[f(x)] = f(\mu) + \frac{1}{2} f''(\mu) \sigma_x^2 \quad (4)$$

The expected value of  $f^2(x)$  is expressed as

$$E[f^2(x)] = f^2(\mu) + \frac{1}{2} [f^2(\mu)]'' \sigma_x^2$$

---

\* Definition of expectation and variance are presented in Appendix A.

$$\begin{aligned}
&\approx f^2(\mu) + \frac{1}{2} [2f(\mu)f'(\mu)]' \sigma_x^2 \\
&\approx f^2(\mu) + \frac{1}{2} [f'(\mu)f'(\mu) + 2f(\mu)f''(\mu)] \sigma_x^2 \\
&\approx f^2(\mu) + [f'(\mu)]^2 + f(\mu)f''(\mu) \sigma_x^2
\end{aligned} \tag{5}$$

The variance of a variable  $x$  is derived as follows:

$$\begin{aligned}
\sigma_x^2 &= V[x] = E[(x - \mu)^2] = E[x^2 - 2\mu x + \mu^2] \\
&= E[x^2] - 2\mu E[x] + [\mu^2]
\end{aligned}$$

as  $E[x] = \mu$  ; and  $E[\mu^2] = \mu^2$  , as  $\mu$  is a constant,

$$\sigma_x^2 = V[x] = E[x^2] - 2\mu^2 + \mu^2 = E[x^2] - [E(x)]^2 \tag{6}$$

In other words, the variance is the mean of the square minus the square of the mean. The variance of  $f(x)$  can be written as

$$V[f(x)] = E[f^2(x)] - E[f(x)]^2 \tag{7}$$

Substituting Equations 4 and 5 into Equation 7 results in

$$\begin{aligned}
V[f(x)] &\approx f^2(\mu) + [f'(\mu)]^2 + f(\mu) f''(\mu) \sigma_x^2 \\
&\quad - f^2(\mu) + f(\mu) f''(\mu) \sigma_x^2 + \frac{1}{4} [f''(\mu)]^2 \sigma_x^4 \\
V[f(x)] &\approx [f'(\mu)]^2 \sigma_x^2 - \frac{1}{4} [f''(\mu)]^2 \sigma_x^4
\end{aligned} \tag{8}$$



10. In Equations 4 and 8, if the random variables can be assumed normally distributed, the second-order terms are small and may be neglected. For multivariate situations, the first-order approximation to the expectation and the variance of  $f(x)$  is expressed by Benjamin and Cornell (1970) as

$$E[f(x)] = f(\mu) \quad (9)$$

$$V[f(x)] = \sum_{i=1}^N \sum_{j=1}^N \left( \left. \frac{\partial f}{\partial x_i} \right|_{\text{all } \bar{x}_i} \left. \frac{\partial f}{\partial x_j} \right|_{\text{all } \bar{x}_j} \right) \text{Cov}(x_i, x_j) \quad (10)$$

where  $\text{Cov}(x_i, x_j)$  is the covariance of variable  $x_i$  and  $x_j$ . Note that if the  $x$ 's are uncorrelated, the covariance is zero, and Equation 10 is simply

$$V[f(x)] = \sum_{i=1}^N \left( \left. \frac{\partial f}{\partial x_i} \right|_{\text{all } \bar{x}_i} \right)^2 V x_i \quad (11)$$

11. To use Equations 4 and 8 to compute the expected value and variance of the function  $f(x)$ , the function should have derivatives of at least the first order; derivatives of the second order are needed only when the nonlinear terms in the equations are considered. An analytical expression (such as the CBR equation in Equation 1) is needed to determine the derivatives of the function. In the layered elastic method, the expected value and variance of the maximum strains are to be determined. However the strains are not determined from any analytical expression but computed using the BISAR computer program. Therefore, other procedures have to be developed in order to use the Taylor series expansion method with the layered elastic method. The Rosenblueth method presented below is readily suited for elastic layered method and the BISAR computer program, and this method is much simpler than the Taylor series expansion. This is particularly true when derivations of higher order are required in the Taylor series expansion method.

# Rosenblueth Method

12. Equations 4 and 8 are obtained from the Taylor series expansion of the function about the expectations of the random variables. This method requires the existence and continuity of the first and second derivatives of the function. Rosenblueth (1975) overcame these difficulties through use of point estimates of the function. The expressions for the expected value are:

$$E[\epsilon^N] = \frac{1}{2} \left( \epsilon_+^N + \epsilon_-^N \right) \quad \text{for one variable} \quad (12)$$

$$E[\epsilon^N] = \frac{1}{2^2} \left( \epsilon_{++}^N + \epsilon_{+-}^N + \epsilon_{-+}^N + \epsilon_{--}^N \right) \quad \text{for two variables} \quad (13)$$

$$E[\epsilon^N] = \frac{1}{2^3} \left( \epsilon_{+++}^N + \epsilon_{++-}^N + \epsilon_{+-+}^N + \epsilon_{+--}^N + \epsilon_{-++}^N + \epsilon_{-+-}^N + \epsilon_{--+}^N + \epsilon_{---}^N \right) \quad \text{for three variables} \quad (14)$$

$$E[\epsilon^N] = \frac{1}{2^4} \left( \epsilon_{++++}^N + \epsilon_{+++ -}^N + \epsilon_{++ - +}^N + \epsilon_{+ - ++}^N + \epsilon_{+ - - +}^N + \epsilon_{- + ++}^N + \epsilon_{- + - +}^N + \epsilon_{- - ++}^N + \epsilon_{- - + -}^N + \epsilon_{- - - +}^N + \epsilon_{- - - -}^N \right) \quad \text{for four variables} \quad (15)$$

$$E[\epsilon^N] = \frac{1}{2^M} \left( \underbrace{\epsilon_{+++++}^N}_M + \cdots + \underbrace{\epsilon_{-----}^N}_M \right) \quad \text{for M variables} \quad (16)$$

Note that the number of total terms to calculate the expected value of a function  $\epsilon$  (strain computed using the BISAR program) which has  $M$  variables is  $2^M$ , and  $N$  has a value of either 1 or 2 as shown in Equation 7, i.e.,  $\epsilon$  is represented by  $f(x)$ , and  $N$  is the power of the function.

13. To reduce the number of variables in layered elastic method computations, variations of Poisson's ratio of pavement materials are neglected, as it has insignificant effect on pavement response to loads. The variation of tire contact area can also be neglected in this computation that was concluded in the previous study of CBR design method for airfield pavements (Chou 1986) and should hold for any rational method of design. To illustrate the use of the Rosenblueth method, the computation of the expected value of the strain  $\epsilon$  for an all-bituminous flexible pavement is presented. The independent parameters considered but uncertain are the wheel load  $P$ , the elastic modulus and thickness of the bituminous layer  $E_1$  and  $h_1$ , respectively, and the elastic modulus of the subgrade  $E_2$ . Other parameters (wheel contact area and Poisson's ratio) are precisely known, i.e., the standard deviations are zero. For a four-parameter problem, Equation 15 is used to determine the expected value of the strain  $\epsilon$ . Assuming that the standard deviations of the parameters are  $\sigma_P$ ,  $\sigma_{E1}$ ,  $\sigma_{E2}$ , and  $\sigma_{h1}$  and that the parameters are arranged in the order of  $P$ ,  $E_1$ ,  $E_2$ , and  $h_1$  (i.e., the order of the symbols + + + +, + + + -, ..., etc.), each term in Equation 15 is computed using the BISAR program in the manner shown in Table 1. Once the mean values for parameters  $\bar{P}$ ,  $\bar{E}_1$ ,  $\bar{E}_2$ , and  $\bar{h}_1$  and their standard deviations  $\sigma_P$ ,  $\sigma_{E1}$ ,  $\sigma_{E2}$ , and  $\sigma_{h1}$  are specified, the expected value of  $\epsilon$  can be determined from Equation 15, and the variance of  $\epsilon$  is computed using Equation 7.

#### Reliability Analysis

14. As soon as the expected value and the variance of a function (such as the strain values computed in a layered elastic pavement system or the  $a$  factor in Equation 2 representing the traffic performance level) are determined, the reliability level of the function can be computed. Reliability is defined as the probability that the pavement system will perform its intended function over its design life (or time) and under the conditions (or environment) encountered during operation (Darter and Hudson 1973). The procedure to follow is explained on the following page.

15. The expected strain value  $\epsilon$  (in each term) in Equations 12 to 16 is computed using the BISAR program. The performance (strain repetitions to failure) of the pavement may be estimated from the failure criteria shown in Figures 1 and 2 for two different failure modes.

16. With the strain value  $\epsilon$  assumed normally distributed, the number of strain repetitions corresponding to  $\epsilon + \epsilon \sigma_{\epsilon}$  (or  $\epsilon[1 + C \cdot CV(\epsilon)]$ ) can be determined from Figures 1 and 2, and the probability of  $\epsilon \leq \epsilon[1 + C \cdot CV(\epsilon)]$  is taken from the normal distribution.  $CV(\epsilon)$  is the coefficient of variation of  $\epsilon$ , which is the ratio of the standard deviation of  $\epsilon$  to a mean of  $\epsilon$ , (i.e.,  $\sigma_{\epsilon}/\bar{\epsilon}$ ), and  $C$  is the selected number varying from -3 to +3.  $C$  values less than -3 and greater than +3 are not necessary because the areas

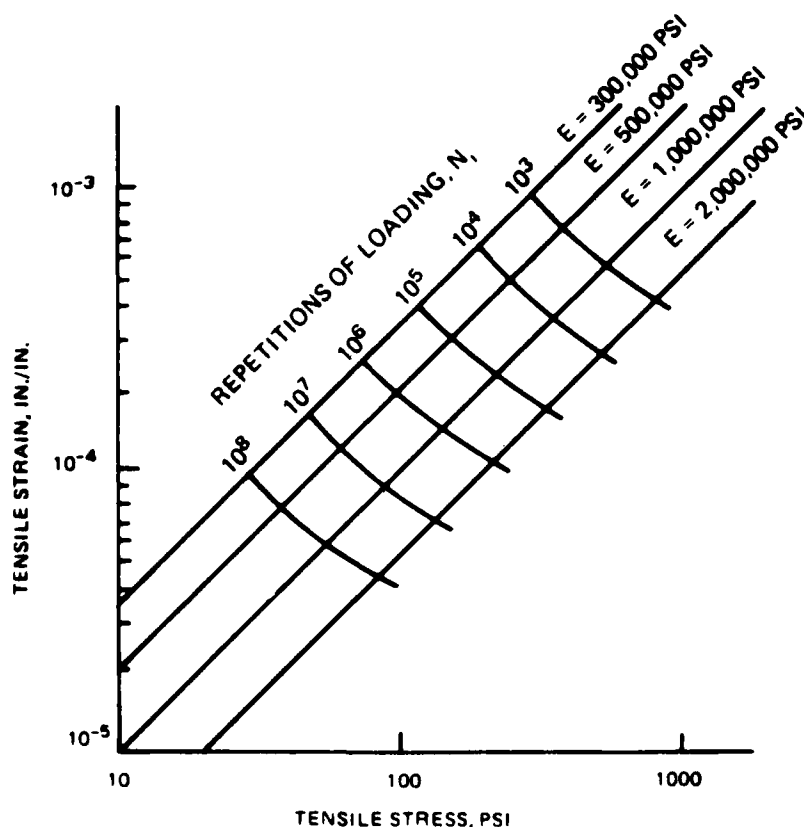


Figure 1. Criteria for limiting horizontal tensile strain in the bituminous concrete (after Heukelom and Klomp 1962)

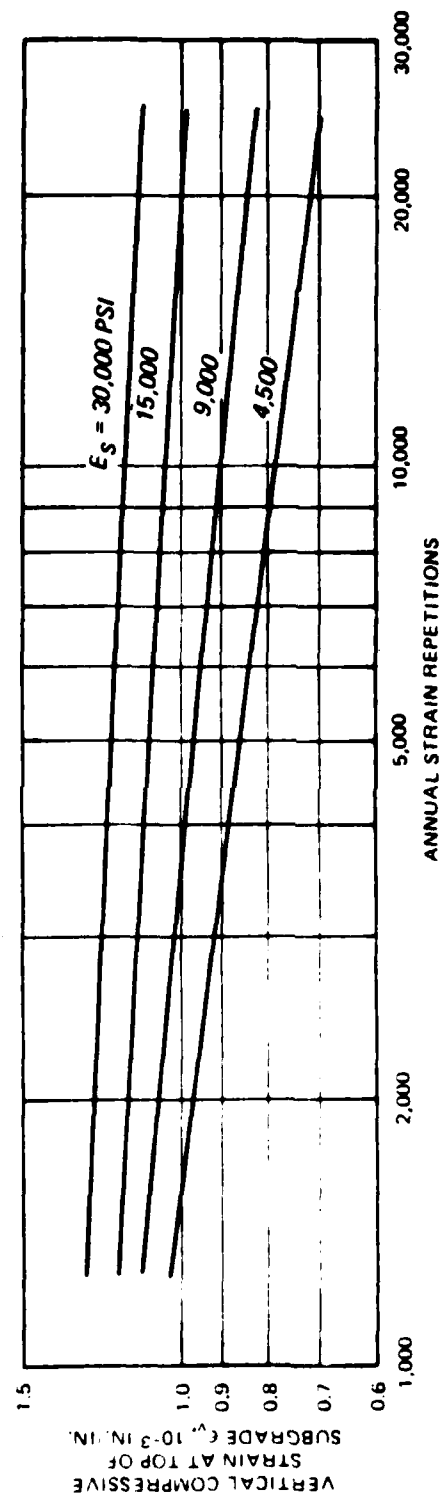


Figure 2. Criteria for limiting vertical compressive strain at the top of the subgrade

under a normal distribution curve beyond -3 and +3 standard deviations are negligible. The computations of the reliabilities, strain values, and strain repetitions for given airfield pavements are illustrated in the design examples in Part V. Table 2 shows the partial results from the RELIBISA computer program for a given airfield pavement with given input parameter variabilities. In this particular example, the strain computed from mean values of input parameters is  $0.31398\text{E-}03$  in./in.\*, and its initial reliability is assumed to be 0.5 (Potter 1985). The standard deviation of the strain  $\sigma_\epsilon$  computed from Equation 7 is  $0.31398\text{E-}03$  in./in. Reliability values are selected from the normal distribution curve between -3 and +3 standard deviations. Strain repetitions are calculated for the specific failure mode from strain values corresponding to different reliability levels.

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\* A table of factors for converting non-SI units of measurement to SI (metric) units is presented on page 3.

#### PART IV: COMPUTER PROGRAM RELIBISA

17. To analyze a flexible pavement in terms of probability and reliability, a computer program RELIBISA was prepared. The program is modified from BISAR computer program (Korinklijke/Shell Laboratorium 1972) which computes stresses, strains, and deflections in a pavement structure for a given set of input data. The modification involves the considerations of input parameter variations in each term of Equations 12 to 16 and of the failure criteria in Figures 1 and 2. The input guide and an example run of the program are presented in Appendix B. The logic and operation of the program are presented in the following paragraphs.

##### Input Data

18. The input data for the RELIBISA computer program are listed below.
- a. Number of wheels and wheel spacings in assembly.
  - b. Wheel loads and the tire contact area.
  - c. Elastic modulus, Poisson's ratio, and thickness for each pavement layer.
  - d. Key word for the search of locations of maximum strains in the pavement.

##### Program Logic

19. Once the aircraft type and gear configurations are input to the program, the locations of the maximum strains in the pavement structure are determined using the mean values of the input parameters (i.e., load, elastic modulus, and layer thicknesses). The same locations can be used to compute the strains for each term in Equations 12 to 16 in order to save computer time. For more accurate results, however, the locations of the maximum strains must be determined separately for each term in Equations 12 to 16. This is because of the change in the input parameters. When the input parameters are changed, particularly layer thicknesses, the location of the maximum strain in the pavement structure will also change. Since the search for the exact locations of maximum strains requires more computer time, the locations of the maximum strains in the pavement are assumed either at the centroid of

the gear assembly or at the location directly beneath one wheel load. This assumption can greatly reduce the computer time and sacrifice little accuracy.

20. Since the use of RELIBISA program requires the repetitive usage of BISAR program to compute the strains for different sets of input parameters (one set represents one term in Equations 12 to 16), the computer time can increase rapidly with increasing number of layers. For an all-bituminous pavement, the input parameters are wheel load  $P$ , elastic moduli of the surface and the subgrade  $E_1$  and  $E_2$ , respectively, and the thickness of the surface layer  $h_1$ . The number of terms in Equation 15 is 16 (i.e.,  $2^4 = 16$ ). Because computations are made for both the surface and subgrade layers, the total number of times the BISAR program is used is doubled to 32. For a three-layer pavement system, the total number of times the BISAR program is used is  $2 \times (2^6) = 128$ . When the locations of maximum strains are to be determined for each term in the equation, the total number of computations will then be doubled to 256. The RELIBISA program is presently programmed for layer number not greater than three. For pavements with layers greater than three the number of layers has to be reduced to three using the concept of composite modulus. This procedure is presented in the example problems in Part V.

21. The failure criteria presented in Figures 1 and 2 are used in RELIBISA program. The criteria for allowable strain repetitions  $N$  for the bituminous concrete presented in Figure 1 can be mathematically expressed as

$$N_{\text{Allowable}}(\text{AC}) = 10^A \quad (17)$$

where

$$A = -5 \log_{10} \epsilon - 2.665 \log_{10} (E_{\text{AC}}) + 2.68$$

$\epsilon$  = maximum horizontal tensile strain at the bottom of the asphaltic concrete layer

$E_{\text{AC}}$  = elastic modulus of the asphaltic concrete, psi

22. The criteria for allowable strain repetition  $N$  for the subgrade given in Figure 2 can be expressed as



$$N_{\text{Allowable}}(\text{subgrade}) = 10,000 \left( \frac{A}{\epsilon_{\text{subg}}} \right)^B \quad (18)$$

where

$$\begin{aligned} A &= 0.000247 + 0.000245 \log_{10} E_{\text{subg}} \\ E_{\text{subg}} &= \text{subgrade modulus, psi} \\ \epsilon_{\text{subg}} &= \text{subgrade strain, dimensionless} \\ B &= 0.0658 (E_{\text{subg}})^{0.559} \end{aligned}$$

Note that the annual strain repetitions are used in Figure 2, and the allowable strain repetition in Equation 18 is based on a 20-year service life.

23. The RELIBISA computer program is prepared based on failure criteria presented in Equations 17 and 18. However, other failure criteria such as those for highway pavements and other design conditions for airfield pavements can also be used in the programs. The replacement is done in the subroutine PERFORMN.

PART V: PROBABILISTIC AND RELIABILITY ANALYSIS  
USING LAYERED ELASTIC METHOD

Flexible Airfield Pavement Design

24. A deterministic design procedure was developed at WES (Barker and Brabston 1975) for airfield flexible pavements using the layered elastic method. This procedure includes conventional, all-bituminous concrete (ABC), and chemically stabilized flexible pavements. The designs are based on analytically determined strain values and experimental and laboratory determined material fatigue strengths. The method can handle in a rational manner the variations in the properties of different pavement materials. In the procedure, the use of the cumulative damage concept permits the consideration of cyclic variation in bituminous materials due to variations in temperatures and the variation in subgrade strength resulting from freeze-thaw cycles. The procedure is now being implemented as one of the options of the Corps' flexible airfield pavement design.

25. Several design examples were presented by Barker and Brabston (1975) illustrating the design procedure of flexible airfield pavements using layered elastic method. The results of two design examples are briefly explained in this report. One is a conventional flexible pavement and the other is an ABC pavement. The designs are for a pavement at Shreveport, La., for 200,000 departures of a 750,000-lb B-747 aircraft over a 20-year design life. Other characteristics of the aircraft and other traffic data required in design were as follows:

- a. Number of wheels in assembly: 4.
- b. Wheel spacing: 44 by 58 in.
- c. Wheel load: 44,531 lb.
- d. Tire contact pressure: 182 psi.
- e. Design traffic: 10,000 annual departures (200,000 total departures over a 20-year design life).
- f. Factor for converting departures to coverages: 1.85.

26. The moduli of the bituminous concrete surface course were determined based on the temperature of each month. Local climatological data (average daily mean air temperature for each month) were used to estimate the design pavement temperature for each month for design based on either bituminous concrete strain or subgrade strain. The relationship between the two

different temperatures is also a function of the thickness of the bituminous concrete used in the design.

27. The procedure to determine modulus values of the base and subbase courses was developed based on the principle that the modulus of the layer is a function of those of the layers below. To provide for variation, the granular layers are divided into sublayers for which the modulus of each sublayer is a function of the sublayer thickness and the modulus of the material below the sublayer. Curves and equations for determining the modulus values of granular materials are provided in Barker and Brabston (1975).

28. The subgrade modulus value is determined from laboratory tests. The design modulus used in the examples was 9,000 psi, and the Poisson's ratio was 0.45. For converting the aircraft departures to strain repetitions, one coverage is equal to one strain repetition for the bituminous concrete strain criterion, one departure is equal to one strain repetition for the subgrade strain criterion in conventional flexible pavements, and one departure is equal to one strain repetition for both bituminous concrete and subgrade strain criteria in ABC pavements.

Problem 1: conventional flexible pavement

29. The conventional thickness design curves are first used to estimate the initial thicknesses. A total pavement thickness of 39 in. is required with a 5 in. bituminous concrete surface course, 12 in. of base course, and 22 in. of subbase course. Cumulative damage theory was not used at first in the analysis. The CHEVIT (Heukelom and Klomp 1962) computer program was used to compute tensile strains in the subgrade and the bituminous concrete surface layer. In computing the subgrade strain, the minimum bituminous concrete modulus estimated for the month of July was used (i.e., the most critical condition for subgrade soil). The computed subgrade strain was smaller than the allowable subgrade strain in Figure 2, and it was concluded that the design pavement section provides adequate protection for the subgrade. In checking for fatigue cracking of the bituminous concrete surface course, the most critical condition for the surface course was first assumed; i.e., it was assumed that all of the traffic for the 20-year design life of the pavement is applied during the single month (January) for which the bituminous concrete modulus is at its maximum. The computed strain in the bituminous concrete

layer was the allowable strain determined in Figure 1, and it was decided that the cumulative damage must be determined for the proposed pavement section.

30. The cumulative damage values were computed for sections of 5-, 7-, and 9-in. surface course thickness. The computation results are presented in Table 3. As shown for conventional flexible pavements, more serious damages in the bituminous concrete layer are caused in the colder months when the bituminous concrete is more brittle (even though the tensile strain in the layer is smaller in the winter months than in the summer months). The computed damages were plotted versus surface course thickness, and a bituminous concrete surface course thickness of 8.6 in. was selected for a cumulative damage value of one. The final design section was a 9-in. bituminous concrete surface course, 12-in. base course, and 18-in. subbase course.

#### Problem 2: ABC pavement

31. To estimate the initial thickness of an ABC pavement, the required thickness for the conventional flexible pavement was divided by the appropriate equivalency factor (1.70) and thus the estimated initial thickness was  $39 \text{ in.} \div 1.7 = 22 \text{ in.}$  The bituminous concrete modulus was also determined for each month. The cumulative damage process was used for sections of 18-, 22-, and 26-in. ABC thickness. The computation results are presented in Tables 4 and 5 for damages of the bituminous concrete and the subgrade, respectively. For both failure modes, more serious damages are caused in the summer months. The computed damages were plotted versus ABC thickness. It was found that the required ABC thickness was 17.2 in. for limiting horizontal tensile strain at the bottom of the bituminous concrete, and it was 21 in. for limiting vertical compressive strain at the top of the subgrade.

#### Probabilistic and Reliability Analysis

32. The two design examples used by Barker and Brabston (1975) for flexible airfield pavements using the layered elastic method presented in the previous section are again used in this section for continuity and to illustrate the procedure of flexible airfield pavements in terms of probability and reliability. The same B-747 aircraft with a wheel load of 44,531 lb was used in the computation. The modulus values of the bituminous concrete are selected based on conclusions derived by Barker and Brabston (1975) and

results presented in Tables 3, 4, and 5. The selections are presented in the following paragraphs.

33. For conventional flexible pavements, the critical period during the year is the colder winter months for the surface bituminous layer as the bituminous concrete becomes more brittle and the warmer summer months for the subgrade as the surface layer becomes less stiffer. Accordingly, a modulus value of 1,000,000 psi and a Poisson's ratio of 0.3 are used in the bituminous concrete strain criterion, and a modulus value of 200,000 psi and a Poisson's ratio of 0.5 are used in the subgrade strain criterion. For ABC pavements, the critical periods for both the bituminous concrete and the subgrade are in the warmer summer months. A concrete modulus value of 250,000 psi and a Poisson's ratio of 0.5 are used in the analysis for both strain criteria. For subgrade soil, a modulus of 9,000 psi and a Poisson's ratio of 0.45 are used in the computation as were used in the design examples (Barker and Brabston 1975).

34. In the analysis of an airfield pavement in terms of probability and reliability, the RELIBISA computer program is used to calculate the allowable load repetitions for a given pavement section for various reliability levels. The computer results are similar to those shown in Table 2 for either bituminous concrete or subgrade failure modes. The input parameter variations are considered by the use of the CV of the parameters which are defined to be the ratio of the standard deviation to the mean value of the parameter. For instance, if the mean gear load is 178,000 lb and the CV of the gear load is assumed to be 10 percent, the standard deviation of the gear load will be 17,800 lb, i.e., 68.3 percent of the time the gear load would lie between 160,200 and 195,800 lb (plus and minus one standard deviation).

35. Relationships between reliability level and the corresponding allowable load repetitions are established for many pavement sections with various input parameter variabilities. The reliability of the existing Corps of Engineers flexible pavement design method is 0.5, i.e. if mean values of input parameters are used in the computation, the chance of success of the design is 50 percent that the computed strain repetition is less than or equal to the designed one. The use of 0.5 is suggested by Potter (1985). However, the procedures presented in the RELIBISA computer program are applicable for any reliability value of the design procedure employed. The results of the computations and analyses are presented in the following paragraphs.

36. Different modulus values and computational procedures are used in the thickness design (Barker and Brabston 1975) and the reliability analysis even though the same design examples and design parameters are employed. The thicknesses and allowable strain repetitions computed for the two different analyses will be different, and comparison of computed results between the two analyses should be conducted with caution.

#### ABC pavements

37. Figure 3 shows the relationships between reliability level and strain repetition for an ABC pavement with varying thicknesses. The original thickness design indicated that the required thicknesses for a 20-year design period is 17.2 and 21.2 in. for the bituminous concrete and subgrade strain criterion, respectively. Curves in Figure 3 are plotted based on computer results similar to those shown in Table 2 for each thickness and for each failure criterion. The CV's ,  $P$  ,  $E_1$  ,  $h_1$  , and  $E_2$  for the input parameters are all assumed to be 10 percent. For a given failure criterion, the curves are generally parallel to each other, except in the area when the reliability is close to one and zero. Since the variability  $E_1$  and  $E_2$  are believed to be greater than 10 percent as were used in Figure 3,  $CV(E_1)$  and  $CV(E_2)$  are assumed to be 0.15 and 0.25, respectively, and the computed results are plotted in Figure 4. It is seen that the shapes of the curves are similar to those shown in Figure 3 for different CV of the input parameters. The significance of the curves is that for a given ABC thickness. The allowable strain repetition to failure is presented at its reliability level. For a 22-in. ABC pavement designed against the subgrade failure, the allowable strain repetition at a reliability level of 0.5 is 740,000 strain repetitions, i.e., the chance of success of the design of this pavement to sustain 740,000 strain repetitions of the B-747 aircraft load before failure is 50 percent. The chance of success can be increased to 80 percent if the design strain repetition is reduced to 105,000. For example, if the 740,000 repetitions are considered as a 20-year design period and the 105,000 repetitions are thus equivalent to 2.8 years, there is an 80 percent chance that the pavement can last 2.8 years without failure (with routine maintenance), but there is only a 50 percent chance that the pavement can last a full 20-year design period. The flatter the slope of the curves in Figures 3 and 4, the greater are the uncertainties involved in the design. However, the steeper the slopes of the curves, the lesser are the uncertainties involved in

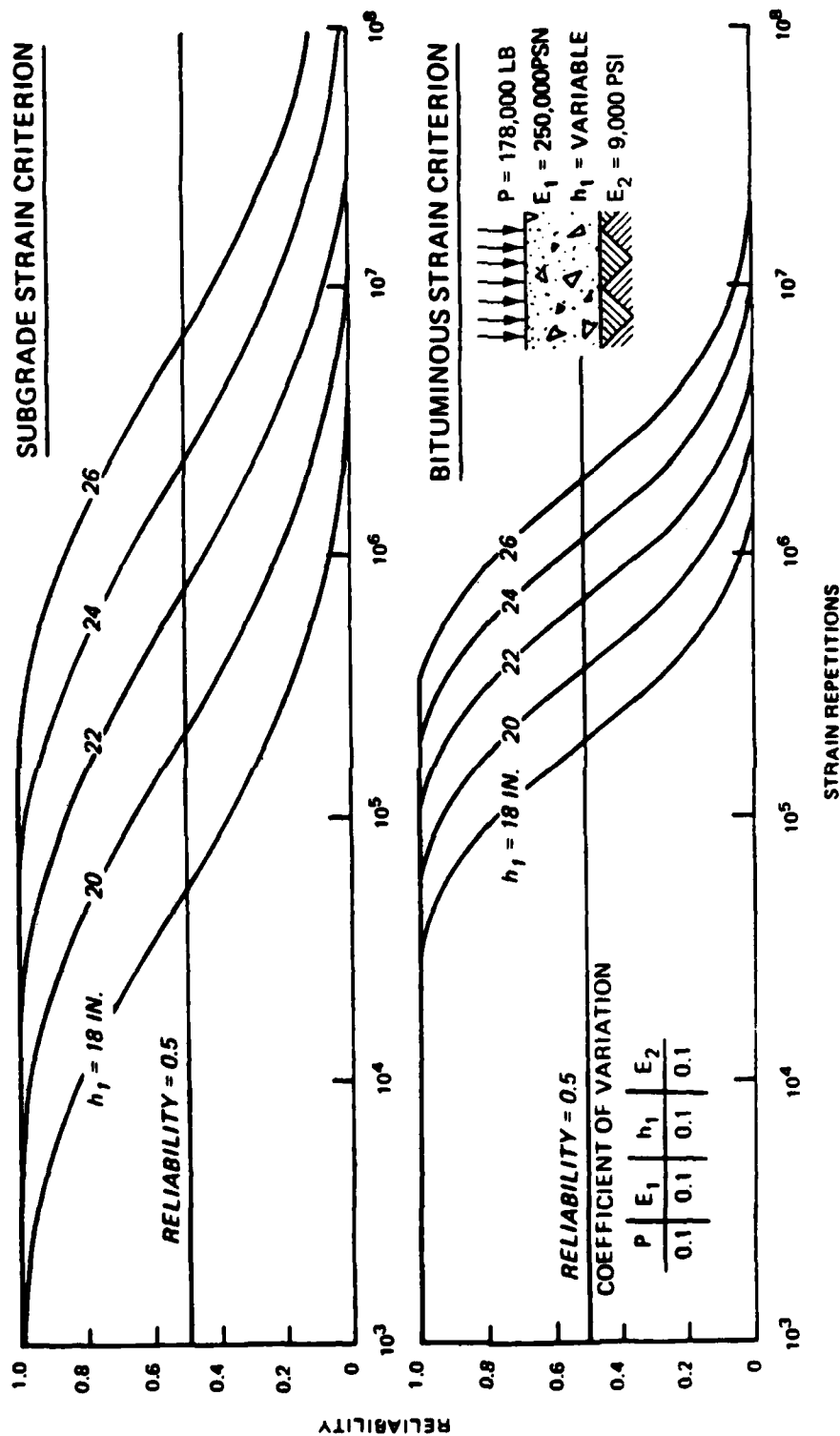


Figure 3. Relationships between reliability and strain repetitions for ABC pavements with the same CV

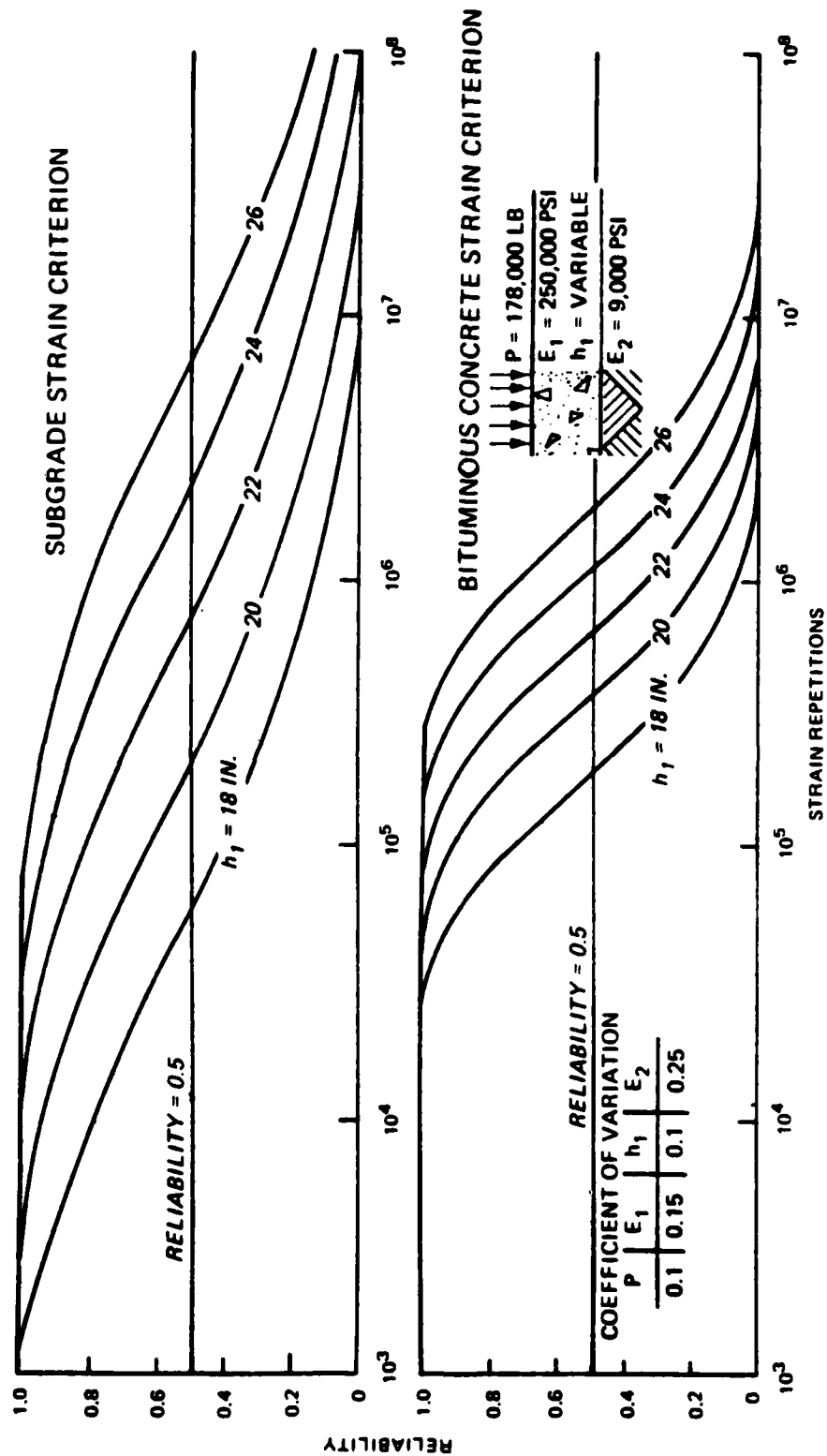


Figure 4. Relationships between reliability and strain repetitions for ABC pavements with varying CV



the design. The shapes of the curves are influenced by the failure criteria (Figures 1 and 2) employed in the computations. This will be discussed in Part VII.

38. The curves in Figures 3 and 4 for the bituminous concrete strain criterion have steeper slopes than those for the subgrade strain criterion, indicating that using the Corps of Engineers' failure criteria (Figures 1 and 2), the designed flexible pavement may have a greater degree of uncertainty in preventing subgrade failure than in preventing fatigue cracking of bituminous concrete surface course.

39. For a given design strain repetition, the relationships between reliability and ABC thickness can also be obtained from Figures 3 and 4. Engineers can choose the ABC thickness suitable for the selected reliability level of the design. Relationships between strain repetition and ABC thickness at different reliability level can also be obtained from Figure 4. The relationships are plotted in Figure 5 which can be helpful to designers in selecting the allowable strain repetitions of a given pavement section for a desired reliability level or to vary the ABC thickness that will be suitable for a specific design performance level of the pavement for a given reliability level. The slope of the curves shown in Figure 5 is the rate of increase of allowable strain repetition because of the increase of pavement thickness for a given reliability level. In Figure 5 the slopes of the curves for the subgrade strain criterion are much steeper than those for the bituminous concrete strain criterion, indicating that at a given reliability level of the design, increasing ABC thickness would increase the allowable strain repetition (or pavement service life) more rapidly with respect to the subgrade failure than would for the fatigue cracking failure of the bituminous concrete. This conclusion, however, can not be used to directly benefit the ABC pavement design using the layered elastic method because the ABC thickness is determined separately for the bituminous concrete and the subgrade strain criteria, and the thicker ABC layer of the two is selected for the design. If the ABC thickness computed for the bituminous concrete failure criterion (designated as  $T_{bitu}$ ) is smaller than that computed for the subgrade failure (designated as  $T_{subg}$ ), the thickness  $T_{subg}$  is selected for the design. The added extra thickness ( $T_{subg} - T_{bitu}$ ) will certainly make the pavement extra safe from fatigue cracking of the bituminous concrete surface course. If the computed thickness  $T_{bitu}$  is larger than the thickness  $T_{subg}$ , the use of

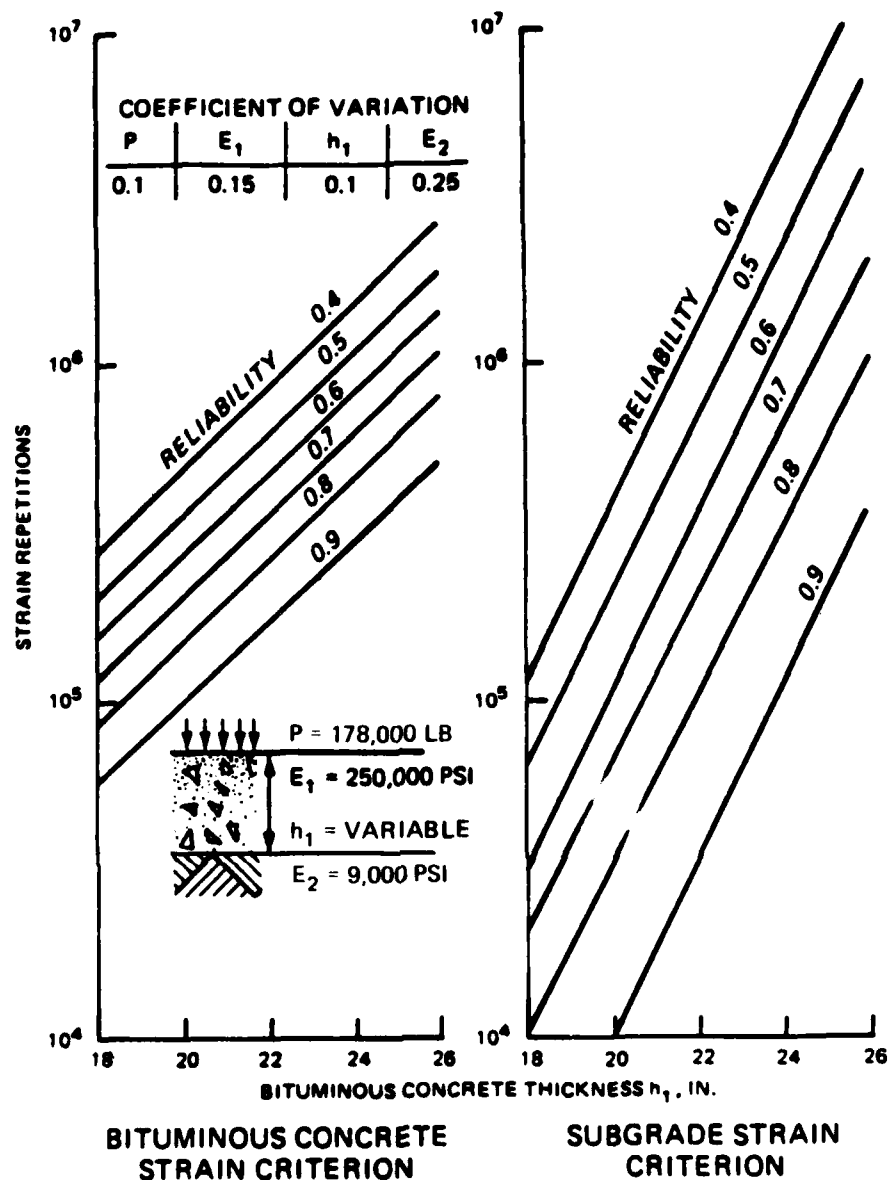


Figure 5. Relationships between pavement thickness and strain repetition (from Figure 4) of ABC pavements

$T_{\text{bitu}}$  for the design can substantially increase the reliability level of the design with respect to the subgrade failure. Also, since the design of ABC pavement using the Corps of Engineers' failure criteria (Figures 1 and 2) results in a design which has a greater degree of uncertainty in preventing subgrade failure, the extra ABC thickness (i.e.,  $T_{\text{bitu}} - T_{\text{subg}}$ ) in the design would certainly increase rapidly the allowable strain repetition against subgrade failure (see Figure 5) and thus increase the reliability and reduce the degree of uncertainty of the design against subgrade failure.

40. The conclusions drawn from Figures 3, 4, and 5 are based on a subgrade modulus of 9,000 psi. Questions arise as to whether a stronger subgrade support of the pavement would reverse the observed trend. Computations similar to those presented in Figure 4 were made for a subgrade modulus of 25,000 psi. It was found that the relationships between the reliability and the strain repetition are very similar to those shown in Figure 4, except that the curves shift to higher strain repetition values because of stronger subgrade support. This is more predominate in the subgrade strain failure mode than in the bituminous concrete failure mode because stronger subgrade support has greater effect on pavement performance with respect to subgrade failure than with respect to bituminous concrete failure.

41. For a good pavement design, it is ideal to have the pavement structure failed in fatigue cracking and subgrade failure at nearly the same traffic level as well as the same reliability level. An optimum ABC thickness may be selected using the relationships plotted in Figure 5. In some cases, however, a subgrade modulus value other than 9,000 psi may be needed to determine the optimum thickness.

42. Figure 4 shows the relationships between the reliability and strain repetition for a given set of CV's and for several ABC thicknesses. Computations were made for a 22-in. ABC pavement with three different sets of CV's for each input parameter having the same CV. The results are plotted in Figure 6. For a CV of 0.01, the variabilities of the input parameters are very small, and the standard deviation for the input parameter, for instance  $h_1 = 22$  in., is 0.22 in. Statistically, 68.3 percent of the designed runway has an ABC thickness of 21.8 to 22.2 in., indicating a design with very small uncertainty and high level of confidence. This is shown in Figure 6 as the curves labeled "AA" have very steep slopes.

43. For the subgrade strain criterion (Figure 6), the AA curve shows that the chance of success is nearly 100 percent the designed 22-in. pavement can last 450,000 strain repetitions and fail after 1,030,000 repetitions. When the CV is increased, i.e., uncertainty about the input parameters increases, the slopes of the curve become flatter as represented by curves BB and CC in Figure 6 for CV of 0.1 and 0.2, respectively.

44. The results presented in Figures 3 and 4 assume that all of the four design input parameters have variations. To study the effect of each individual parameter on pavement performance, computations were made to vary

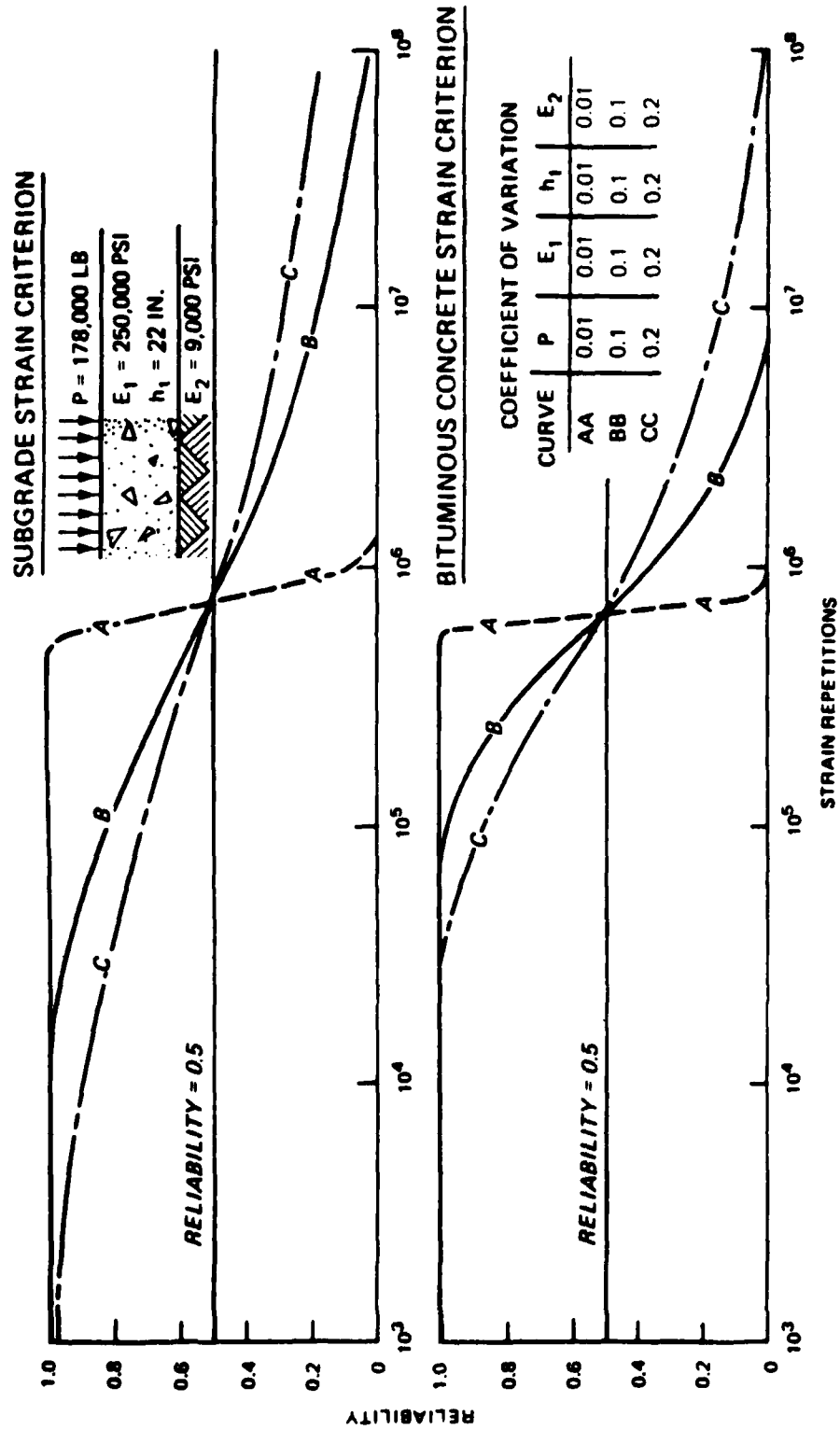


Figure 6. Relationships between reliability and allowable strain repetitions for different sets of CV

only one parameter each time while the variations of the other three parameters were zero. The results are plotted in Figures 7 and 8 for the CV of 0.1 and 0.3, respectively. As discussed, the significance of the slopes of the curves is that the steeper the slope, the lesser the pavement performance is sensitive to the variation of the parameter. Figures 7 and 8 show that for both failure criteria, the pavement performance (allowable strain repetition) is least sensitive to the variation of the subgrade modulus  $E_2$  and followed by the variation of the bituminous concrete modulus  $E_1$ . The pavement performance is most sensitive to the variation of the bituminous concrete thickness  $h_1$  and is followed by the aircraft gear load  $P$ .

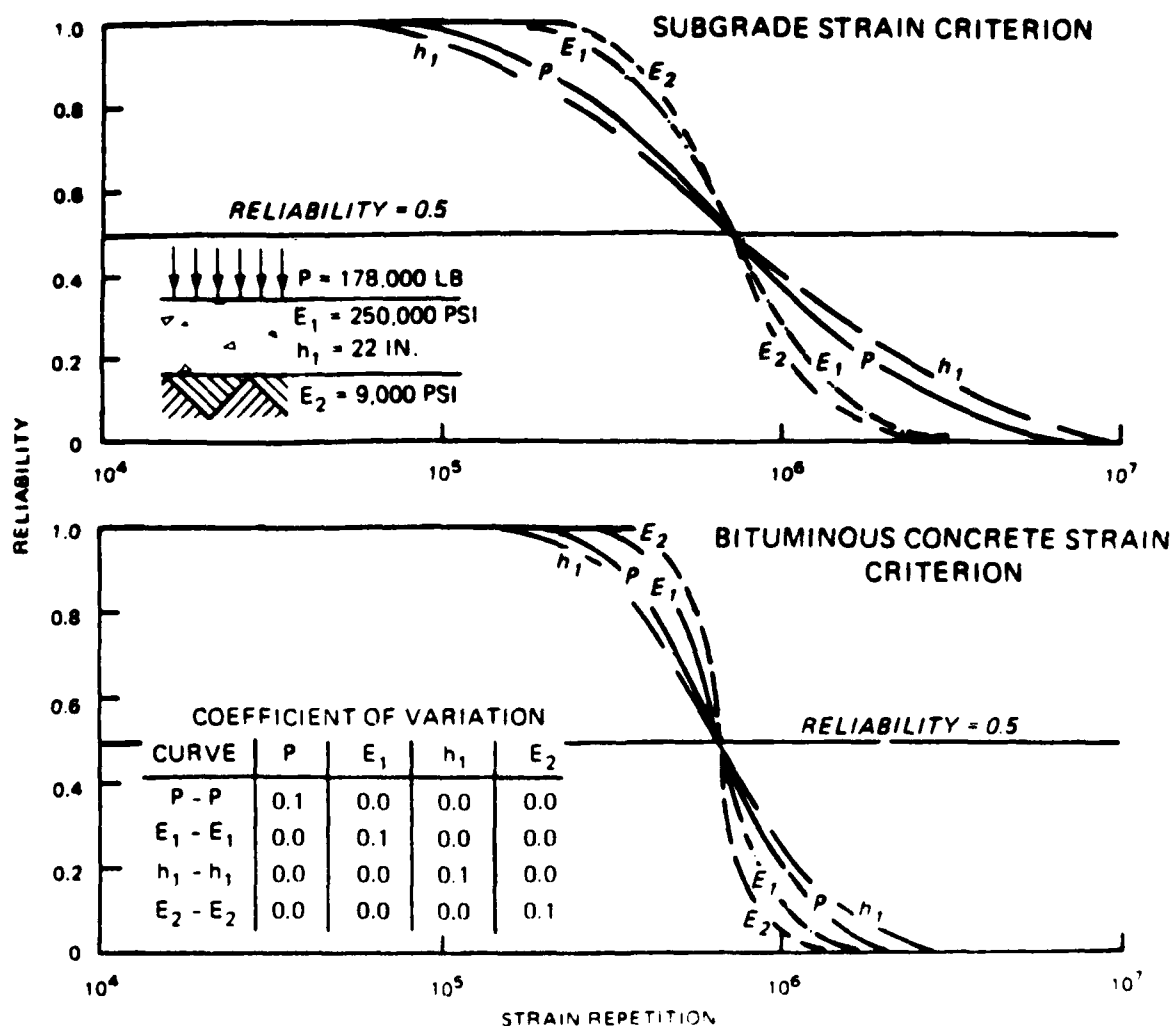


Figure 7. Relationships between reliability and allowable strain repetition for ABC pavements with varying CV,  $10^{-4}$  -  $10^{-7}$

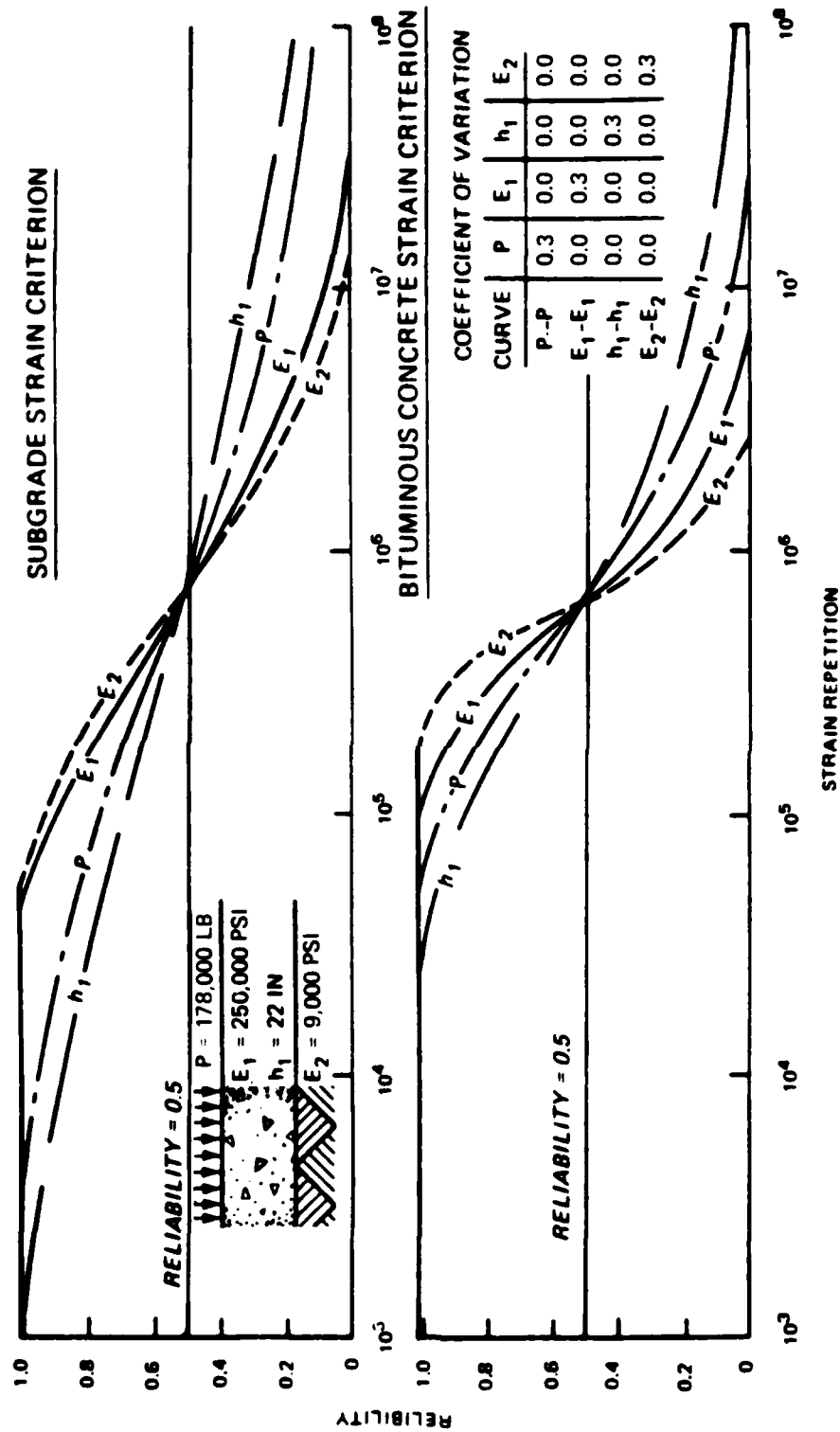


Figure 8. Relationships between reliability and allowable strain repetition for ABC pavements with varying CV

45. The significance of the results presented in Figure 7 may also be explained from another view by using the values listed in Table 6. Table 6 shows the ranges of computed allowable strain repetitions within  $+1$  and  $-1$  standard deviation of the subgrade strain value for four different cases. In each case the CV of one parameter is equal to 0.1, and the CV of the other three parameters is set at zero. The subgrade strain value computed for the pavement in this particular case is 0.000823 in./in. Since the area within  $+1$  and  $-1$  standard deviation under a normal distribution curve is 0.683, the significance of the strain repetitions shown in Table 6 can be explained in the following manner.

46. If only the variation of the bituminous concrete thickness  $h_1$  is accounted for ( $CV(P) = 0.1$ ), there is a 68.3 percent chance that the predicted performance of the pavement falls within the range of 202,718 to 2,889,570 strain repetitions. If only the variation of the subgrade modulus  $E_2$  is accounted for ( $CV(E_2) = 0.1$ ), then for the same 68.3 percent of chance, the predicted performance will fall within a range from 439,127 to 1,156,346 strain repetitions that indicate a smaller variation. A smaller range of predicted pavement performance indicates that the design has a greater chance of success.

47. The results presented in Figure 7 are for CV of 0.1; similar computations were made for CV of 0.3. The results are plotted in Figure 8. The shapes of the curves are similar to those in Figure 7, except that the slopes are much flatter, indicating that because of greater variabilities in the input parameters the designs have a greater amount of uncertainty and thus a smaller chance of success.

48. The results presented in Figures 7 and 8 indicate the variations of bituminous concrete thickness  $h_1$  have the largest effect on pavement performance and is followed by the gear load  $P$ , the bituminous concrete modulus  $E_1$ , and the subgrade modulus  $E_2$ . The significance of this conclusion will be discussed later with the finding for conventional flexible pavements. Also, the results presented in Figures 7 and 8 and Table 6 are for the case that the input parameters have the same CV. In reality, the variation of subgrade modulus  $E_2$  is much larger than the variation of ABC thickness  $h_1$  as the latter can be controlled easier during construction. These variations will be discussed in Part VI.

### Three-layer flexible airfield pavement

49. In the flexible pavement thickness design using the layered elastic method (Barker and Brabston 1975), the granular base and subbase layers are divided into many sublayers, and the elastic modulus of each sublayer is determined based on the moduli of the underlying layers. However, this practice is not suitable in the RELIBISA program because of the requirement for extremely long computer time as the number of input parameter increases. Equivalent modulus values have to be used for the base and subbase layers in the analysis. This is explained in the next paragraph.

50. A typical flexible airfield pavement was used which has a 9-in. bituminous concrete surface layer and a 30-in. granular base course. A modulus of 200,000 psi and a Poisson's ratio of 0.5 were assumed for the surface course for the summer temperatures, and a modulus of 1,000,000 psi and a Poisson's ratio of 0.3 were assumed for the winter temperatures. For the subgrade soil, a modulus of 9,000 psi and a Poisson's ratio of 0.45 were used. Two sets of computations were conducted using the BISAR computer program (Koninklijke/ Shell Laboratorium 1972). In the first set, the 30-in. base course was subdivided into five sublayers, and the modulus values in each layer were determined using the procedures outlined in Barker and Brabston (1975). Strains at the bottom of the surface layer and at the top of the subgrade were computed for both the summer and winter conditions. In the second set of computations, a range of modulus values was assumed for the 30-in. base course which is not subdivided, and the modulus values were determined in such a manner that the computed strain values were matched in the two sets of computations. For the summer temperatures, the determined modulus of the 30-in. granular base course is 60,000 psi when the tensile strains at the bottom of the bituminous concrete layer are equal in the two sets of computations, and the modulus is 33,000 psi when the vertical strains at the subgrade surface are equal. For the winter temperatures, the corresponding moduli are 52,000 psi and 30,000 psi, respectively. For simplicity, average modulus values for the granular material were assumed to be 32,000 psi for the summer temperatures and 55,000 psi for the winter temperatures. These are the modulus values used in all the analyses presented in this section.

51. Two series of computations were made using the RELIBISA computer program to analyze flexible airfield pavement parameters in terms of probability and reliability. In the first series, the thickness of the bituminous



concrete surface course  $h_1$  is held 9 in., and a range of granular base layer  $h_2$  is assumed. In the other series, the thickness of the base layer  $h_2$  is held constant 30 in., and a range of surface layer  $h_1$  is assumed. The CV of the input parameters, gear load  $P$ , moduli of the bituminous concrete  $E_1$ , the granular base course  $E_2$  and the subgrade  $E_3$ , and the thicknesses of the surface course  $h_1$  and the base course  $h_2$  were assumed to be 0.1, 0.15, 0.2, 0.25, 0.1, and 0.15, respectively. The computed results are similar to those in Table 2 and are plotted in Figures 9 and 10 for the subgrade strain criterion (summer temperatures) and the bituminous concrete strain criterion (winter temperatures), respectively.

52. The curves shown in Figure 9 for the subgrade strain criterion are generally parallel to each other. For a given bituminous concrete thickness  $h_1$ , increasing the thickness of base course  $h_2$  can increase the allowable strain repetition of the pavement. This is also true if this procedure is reversed. Figure 10 shows that for the bituminous concrete strain criterion, the performance of the pavement can certainly be improved with the increase of the thickness of the bituminous concrete surface layer  $h_1$  (for a given thickness of the granular base course). This is also true if the thickness of the surface layer  $h_1$  is held a constant and the thickness of the granular base course  $h_2$  is varied, but the benefit reduces rapidly for very thick base course layer, which is demonstrated by the closely spaced curves at greater  $h_2$  values presented in the lower part of Figure 10. The results in Figures 9 and 10 can also be explained by the relationships between the strain repetition and layer thicknesses  $h_1$  and  $h_2$  at different reliability levels plotted in Figure 11. The results presented in Figure 11 are similar to those in Figure 5 for the ABC pavements. The slopes of the curves indicate the rate of change of allowable strain repetition due to change of layer thicknesses  $h_1$  or  $h_2$ . Obviously the steeper the slope of the curve, the better the design for that particular failure mode would be.

53. The curves in Figure 11 in the region where the base course thickness  $h_2$  is less than 30 in. have generally the same slope as those in Figure 11. As the thickness of the base course continues to increase, the slopes of the curves drop slightly. Since the unit cost of granular base course is much less than the bituminous concrete surface course, it is economically more beneficial to increase the thickness of the granular base course to prevent

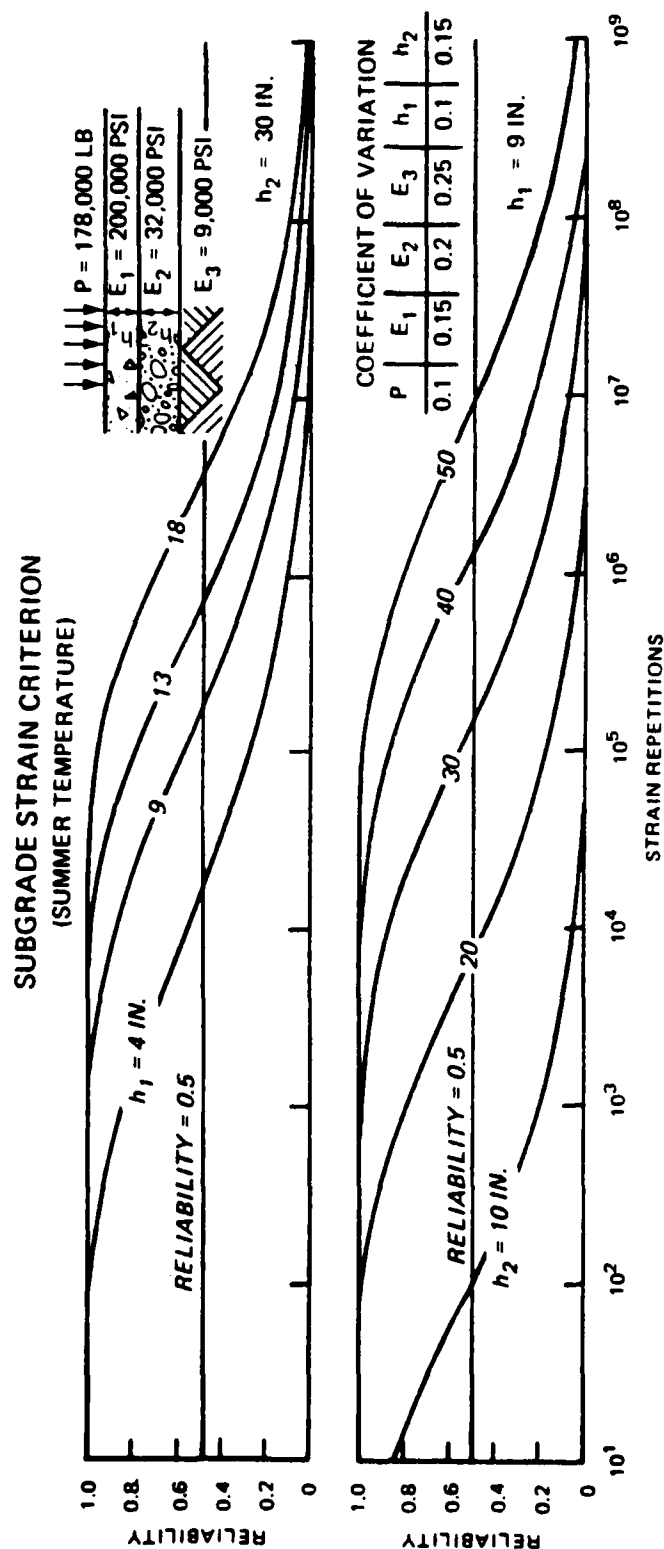


Figure 9. Relationships between reliability and allowable strain repetition for flexible airfield pavements with subgrade strain criterion

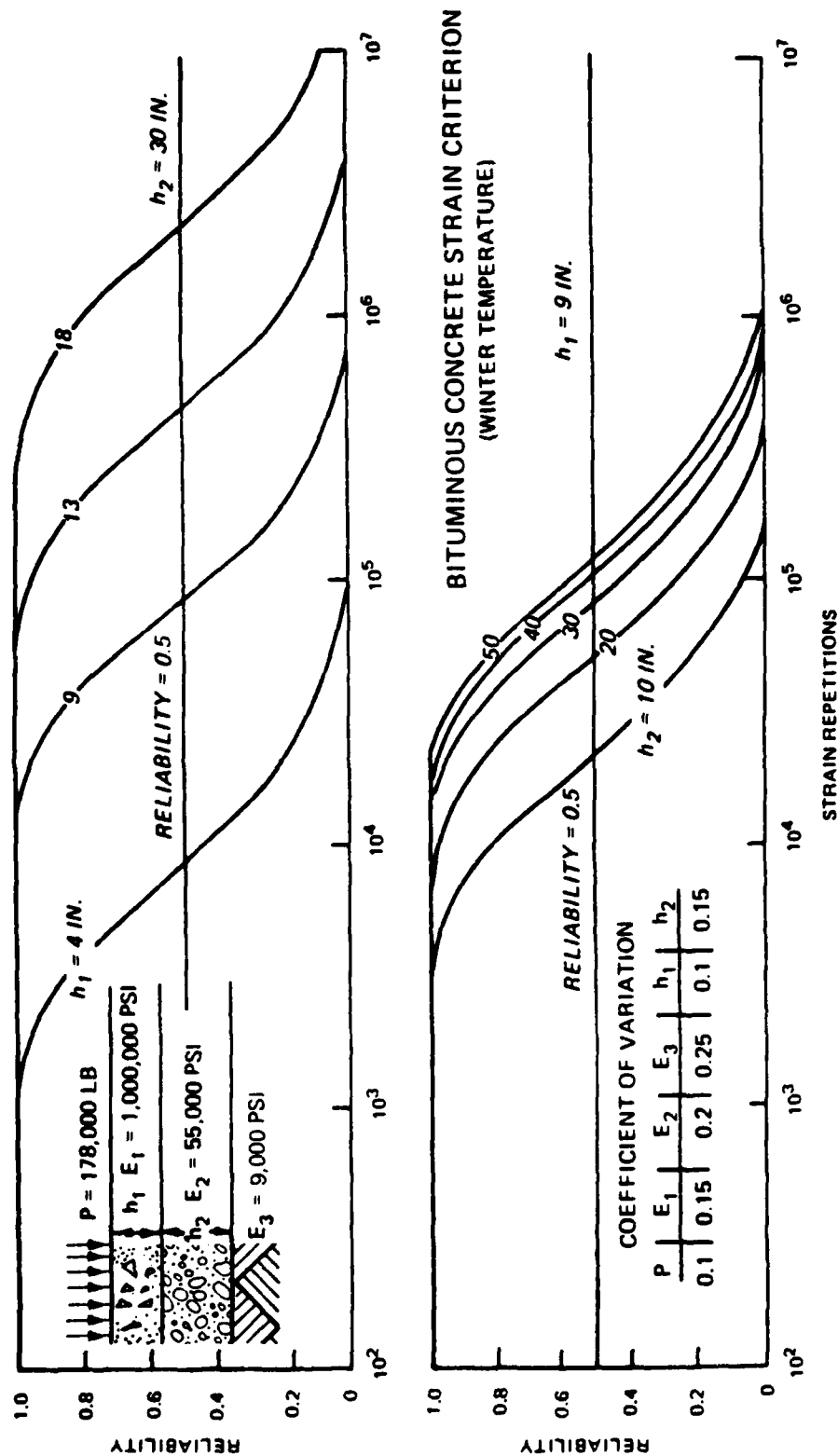


Figure 10. Relationships between reliability and allowable strain repetition for three-layer flexible airfield pavements with bituminous concrete strain criterion

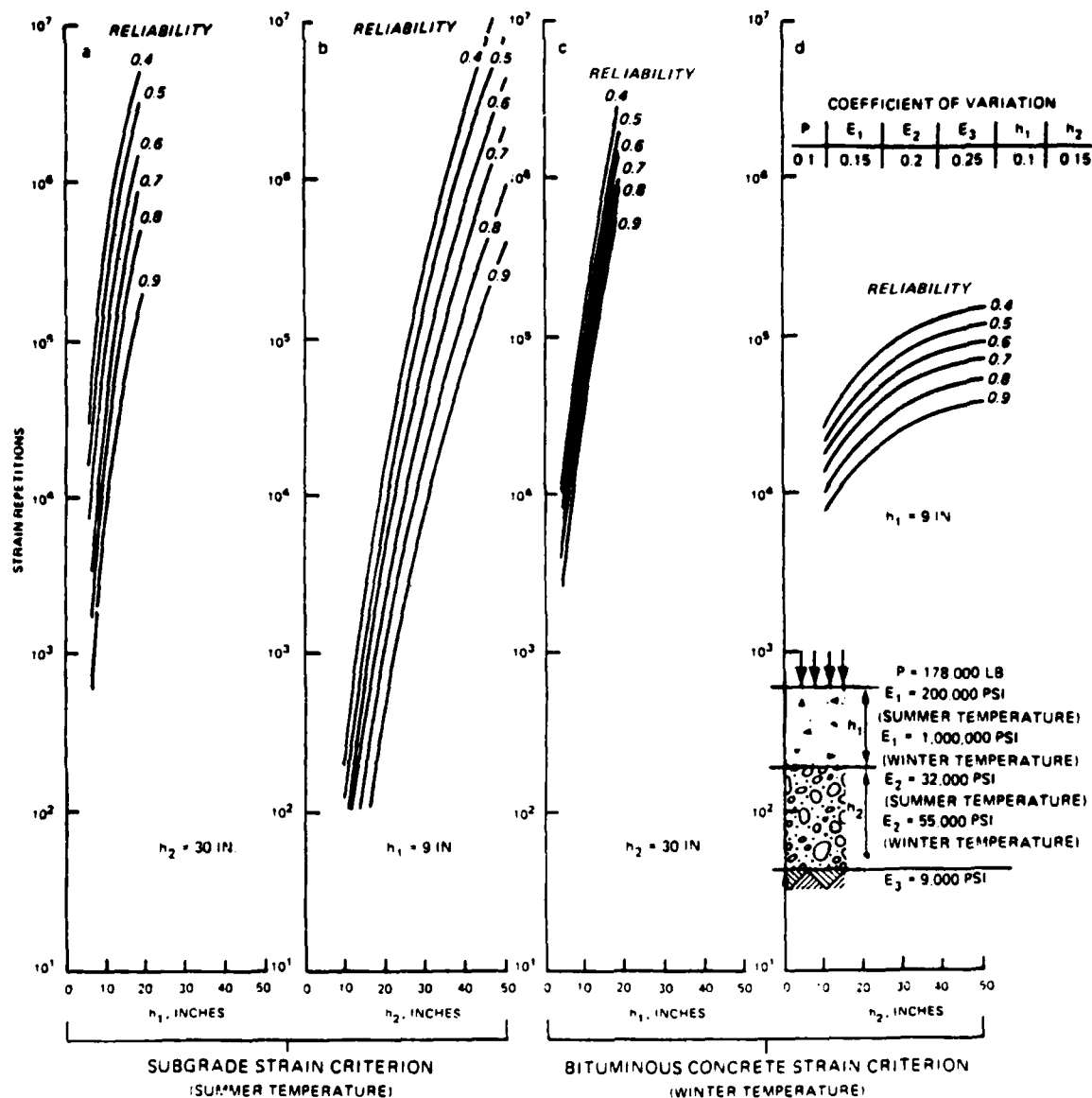


Figure 11. Relationships between pavement layer thicknesses and strain repetition (from Figures 9 and 10) with flexible pavements

the pavement from subgrade failure. This is logical in the structural view-point as the base course is placed directly on the subgrade.

54. Figure 11 shows curves plotted for the bituminous concrete strain criterion. The steep slopes in Figure 11c indicate the structural benefit of increasing bituminous concrete thickness to prevent the pavement from failure of surface cracking. The flatter slopes of the curves in Figure 11d indicate relatively little significance of base course support in a flexible pavement

in the limitation of fatigue cracking of the surface course. Figure 11 also shows that the significance of base course reduces rapidly as its thickness continues to increase.

55. Based on the discussions presented in previous paragraphs, it is more beneficial to increase the granular base course thickness than to increase the bituminous concrete surface course thickness, provided that an adequate thickness of bituminous concrete surface course is used to prevent surface course fatigue cracking.

56. The curves presented in Figure 11 provide the engineers with a tool to vary the layer thicknesses ( $h_1$  or  $h_2$ ) to be suitable for the specific design performance level of the runway and for a given predetermined reliability level of the design. An optimum design may be made to select the thicknesses of the bituminous concrete and the base layers so that the pavement is failed in fatigue cracking and subgrade failure at nearly the same traffic level and the same reliability level.

57. Similar to results presented in Figures 7 and 8 for ABC pavements, Figures 12 and 13 show results of the effect of each individual parameter on the performance of flexible pavements. Computations were made to vary only one parameter each time while variations of the other five parameters were zero. The results presented in Figures 12 and 13 are for the CV's of 0.1 and 0.3, respectively. Both figures show that the variabilities of different input parameters have different degree of impact on pavement performance. For both failure criteria, the pavement performance (allowable strain repetition) is most sensitive to the variation of the aircraft load  $P$ . For the subgrade strain criteria, the pavement performance is more sensitive to variations of the thickness of the granular base layer  $h_2$  (which is placed directly on the subgrade) and the modulus of the subgrade  $E_3$ . The pavement performance is less sensitive to the thickness and modulus of the bituminous concrete surface course  $h_1$  and  $E_1$ , respectively, and the modulus of the base course  $E_2$ . For the bituminous concrete strain criterion, the pavement performance is more sensitive to variations of the thickness and modulus of the bituminous concrete layer  $h_1$  and  $E_1$ , respectively, and is less sensitive to variations of the thickness and modulus of the base layer  $h_2$  and  $E_2$ , respectively, and the modulus of the subgrade  $E_3$ . It is interesting to note that the pavement performance is more sensitive to variations of layer thickness than the elastic modulus of the layer material, and it is least sensitive to the

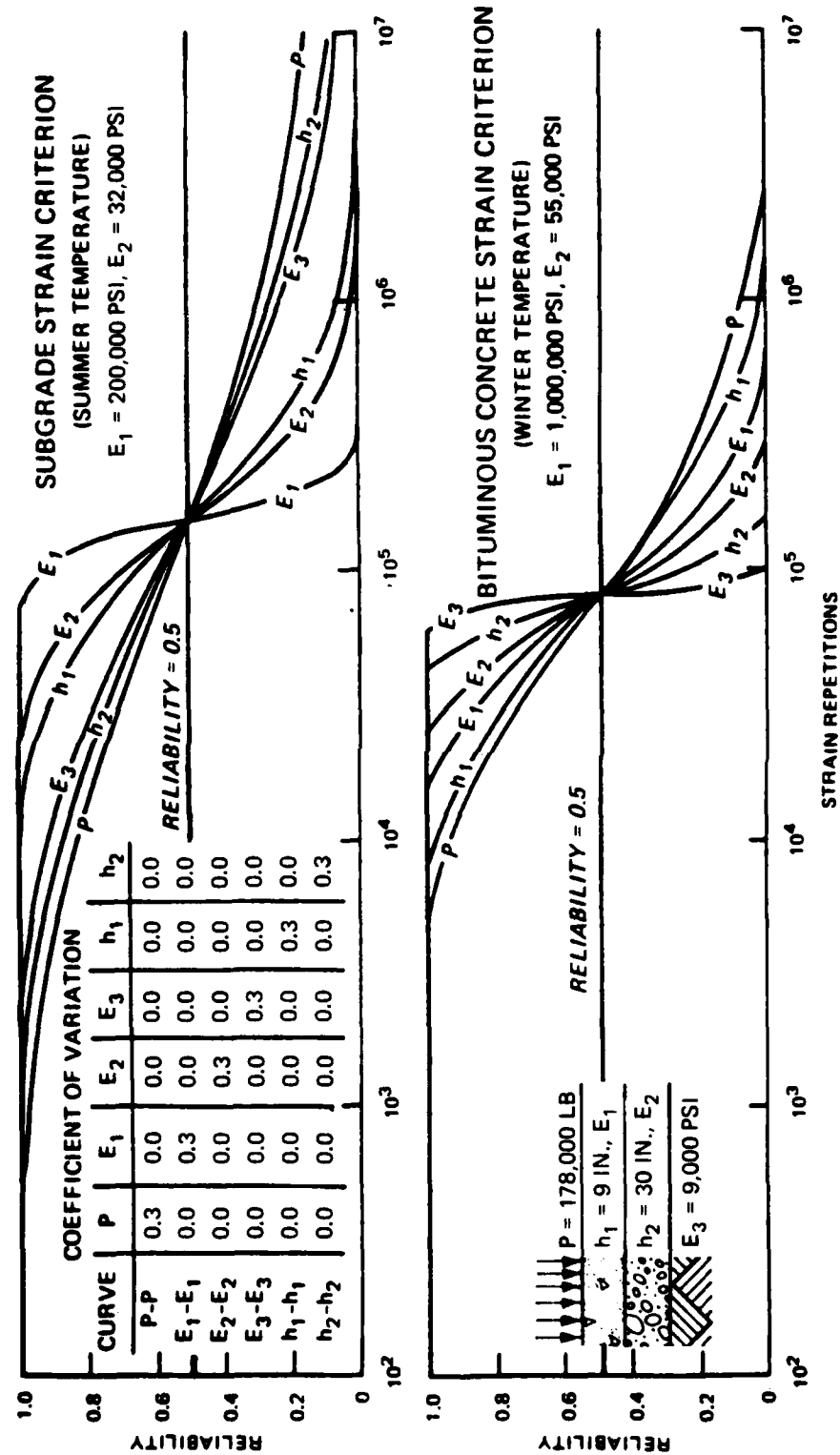


Figure 12. Relationships between reliability and strain repetition for flexible pavements with varying CV

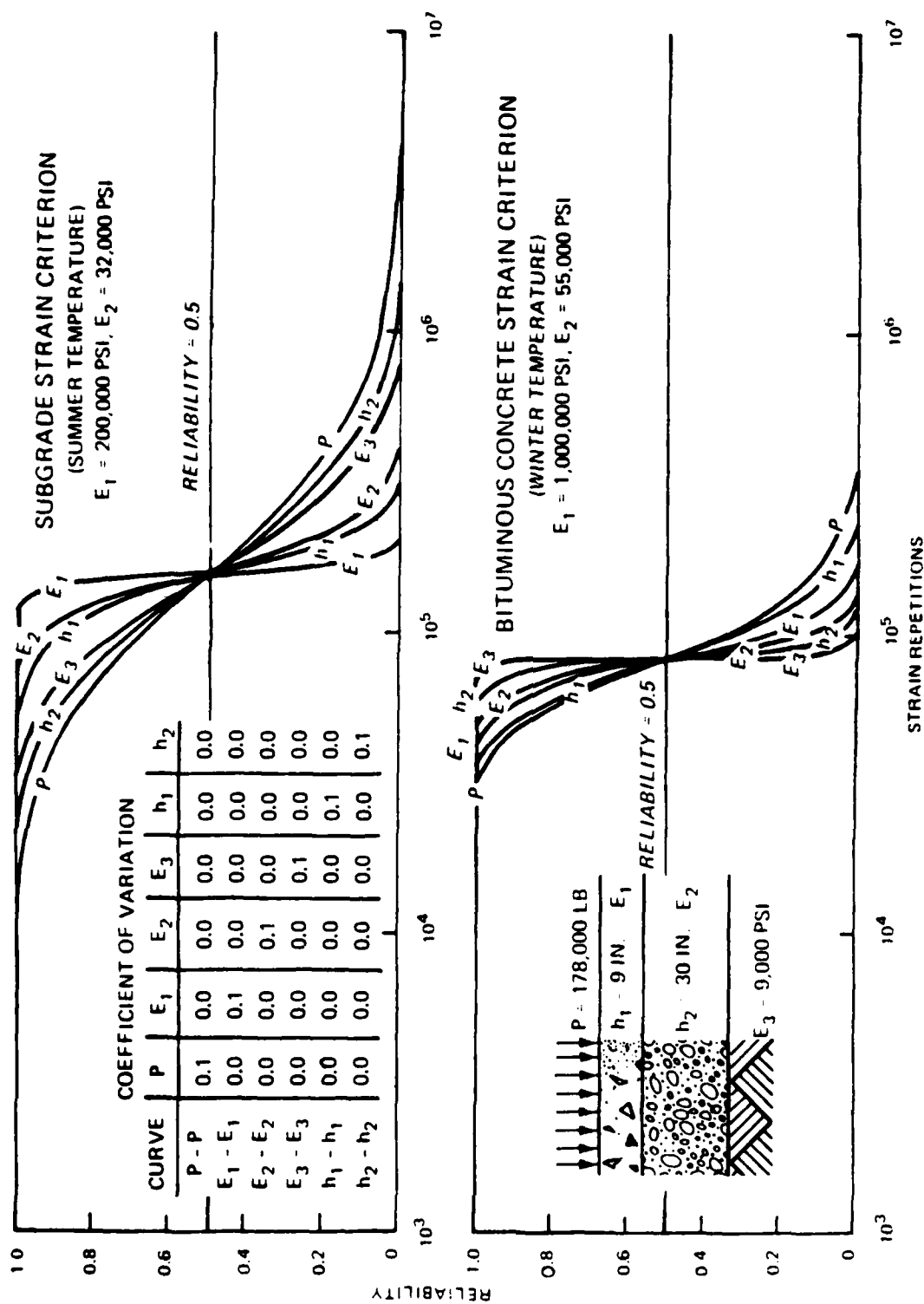


Figure 13. Relationships between variability and strain repetition for flexible pavements with varying CV

variation of modulus of the bituminous concrete surface layer  $E_1$  in the subgrade strain criterion and to the variation of subgrade modulus  $E_3$  in the bituminous concrete strain criterion. Thus, the performance of a flexible pavement is sensitive in the descending order, to variations of  $P$ ,  $h_2$ ,  $E_3$ ,  $h_1$ ,  $E_2$ , and  $E_1$  in the subgrade strain criterion, and to variations of  $P$ ,  $h_1$ ,  $E_1$ ,  $E_2$ ,  $h_2$ , and  $E_3$  in the bituminous concrete strain criterion.

58. The significance of the results presented in Figure 12 may also be explained from another viewpoint by using the values listed in Table 7. Table 7 shows the ranges of computed allowable strain repetitions within +1 and -1 standard deviation of the subgrade strain value for six different cases. In each case, the CV of one parameter is equal to 0.1, and the CV's of the other five parameters are set at zero. The subgrade strain value computed for the pavements is 0.0009404 in./in. Table 7 shows that the standard deviation of the subgrade strain is the largest for the load  $P$  and is the smallest for the modulus of bituminous concrete surface course  $E_1$ . When only the variation of the load  $P$  is accounted for, there is a 68.3 percent chance (i.e., the area within +1 and -1 standard deviation under a normal distribution curve) that the predicted pavement performance falls within the range of 56,160 to 479,000 strain repetitions. If only the variation of the modulus of bituminous concrete  $E_1$  is accounted for, the predicted performance for the same percent of chance narrows down to a range from 142,500 to 169,670 strain repetitions, indicating a smaller variation and thus a design with less uncertainty.

59. The curves presented in Figure 13 are for a CV of 0.3. The patterns of the curves are similar to those in Figure 12, except that the slopes of the curves are much flatter due to larger CV of the input parameters. The flatter slope of the reliability-strain repetition curve indicates greater uncertainties of the designed pavement.

60. Although the pavement performance is more sensitive to the variations of layer thickness than the modulus of the layer, the actual variation of material moduli in the field is known to be larger than the variation of layer thicknesses. This aspect of material variabilities is presented in Part VI.



# PART VI: VARIABILITY OF INPUT PARAMETERS OF FLEXIBLE AIRFIELD PAVEMENTS

61. Figure 5 indicates that for an ABC pavement the pavement performance is sensitive to variations of (in the descending order) ABC thickness  $h_1$ , aircraft load  $P$ , ABC modulus  $E_1$ , and subgrade modulus  $E_2$  for both bituminous concrete and subgrade strain criteria. Figures 12 and 13 indicate that for conventional flexible pavements the performance is sensitive to variations of (in the descending order) load  $P$ , bituminous concrete surface course thickness  $h_1$  and modulus  $E_1$ , respectively, granular base course modulus  $E_2$  and thickness  $h_2$ , respectively, and subgrade modulus  $E_3$  for the bituminous concrete strain criterion, and is sensitive to variations of (in the descending order)  $P$ ,  $h_2$ ,  $E_3$ ,  $h_1$ ,  $E_2$ , and  $E_1$  for the subgrade strain criterion. However, these conclusions are based on the analysis assuming that the input parameters have the same degree of variation. In reality, some parameters have larger variations than others. This section presents the existing knowledge of variability of input parameters for flexible airfield pavements.

62. It has been found that the thickness variations in actual field constructions are not very large. Sherman (1971) measured pavement thicknesses in highway construction in California from 1962 to 1969. Table 8 presents the thickness variations for various pavement materials. Approximate standard errors obtained from the table by pooling the mean squares are shown below (Darter and Hudson 1973):

<u>Material</u>	<u>Standard Error, in.</u>	<u>Number of Tests</u>
Asphalt concrete	0.41	9,775
Cement-treated base	0.68	9,749
Aggregate base	0.79	8,053
Aggregate subbase	1.25	10,578

63. Although the average CV's for these pavement component layers are generally near or less than 10 percent, effort should be made to reduce pavement thickness variation during construction as much as possible.

64. Although the effect of the variation of moduli of layer materials on pavement performance is not as large as that of layer thickness, the actual

variations of moduli in the field are known to be very large. The CV can be as much as 50 percent or more. More efficient construction methods and equipments should be used, and strict compaction and quality controls should be exercised in construction to reduce material modulus variations. Table 8 presents variability recommendations for pavement materials as suggested by Witczak, Uzan, and Johnson (1983) which may be used for different levels of inherent variability.

65. The control of load variation is beyond the jurisdiction of pavement engineers. Since the variation of aircraft load has a large effect on pavement performance, the airfield operators should be informed and advised to limit aircraft overload cases.

## PART VII: SIGNIFICANCE OF FAILURE CRITERIA

66. The relationships between reliability level and strain repetitions presented in Figures 3, 4, 6, 7, 8, 9, 10, 12, and 13 are computed using the RELIBISA computer program which incorporates the failure criteria in Figures 1 and 2. The reliability-strain repetition curves are thus solely determined by the failure criteria incorporated in the computation. To illustrate this point, efforts were made to employ different failure criteria and establish corresponding relationships between reliability and strain repetition.

67. Figure 14 shows four different subgrade strain criteria as represented by criteria A, B, C, and D. The original subgrade strain criterion for a subgrade modulus  $E_s = 9,000$  psi is shown in the figure for comparison. In Figure 14, the three straight lines meet at the point where the strain repetition is 700,000 which is set arbitrarily. Criterion A has a slope steeper than the original criterion. Since the curves are pivoted at 700,000 repetitions, criterion A is more conservative for repetitions less than 700,000 and is less conservative for repetitions greater than 700,000. For instance, at a strain value of  $1.0 \times 10^{-3}$  in./in., the original criterion predicts a strain repetition of 85,000, and criterion A gives a repetition of 3,200,000. Criterion B has a flatter slope than the original and is thus less conservative for repetitions less than 700,000 and more conservative for repetitions greater than 700,000. Criteria C and D are drawn parallel to the original criterion with criterion C above the original and criterion D below. Therefore criterion C gives more conservative predictions, and criterion D gives less conservative predictions than the original criterion predicts.

68. Figure 15 presents the relationships between the reliability and strain repetition computed from the strain criteria in Figure 14. The steeper the slope of the failure strain criterion (Figure 14), the steeper the slope of the reliability-strain repetition curve (Figure 15). The curves in Figure 15 are generally parallel to each other as the corresponding strain criteria in Figure 14. Criterion C in Figure 14 is drawn above the original strain criterion (i.e., more conservative with higher predicted strain repetition for a given strain value), and the reliability-strain repetition curve in Figure 15 (criterion C) is moved to locations of higher strain repetitions than the curve computed from the original strain criterion. The two curves are generally parallel to each other. Criterion D in Figure 14 is drawn below

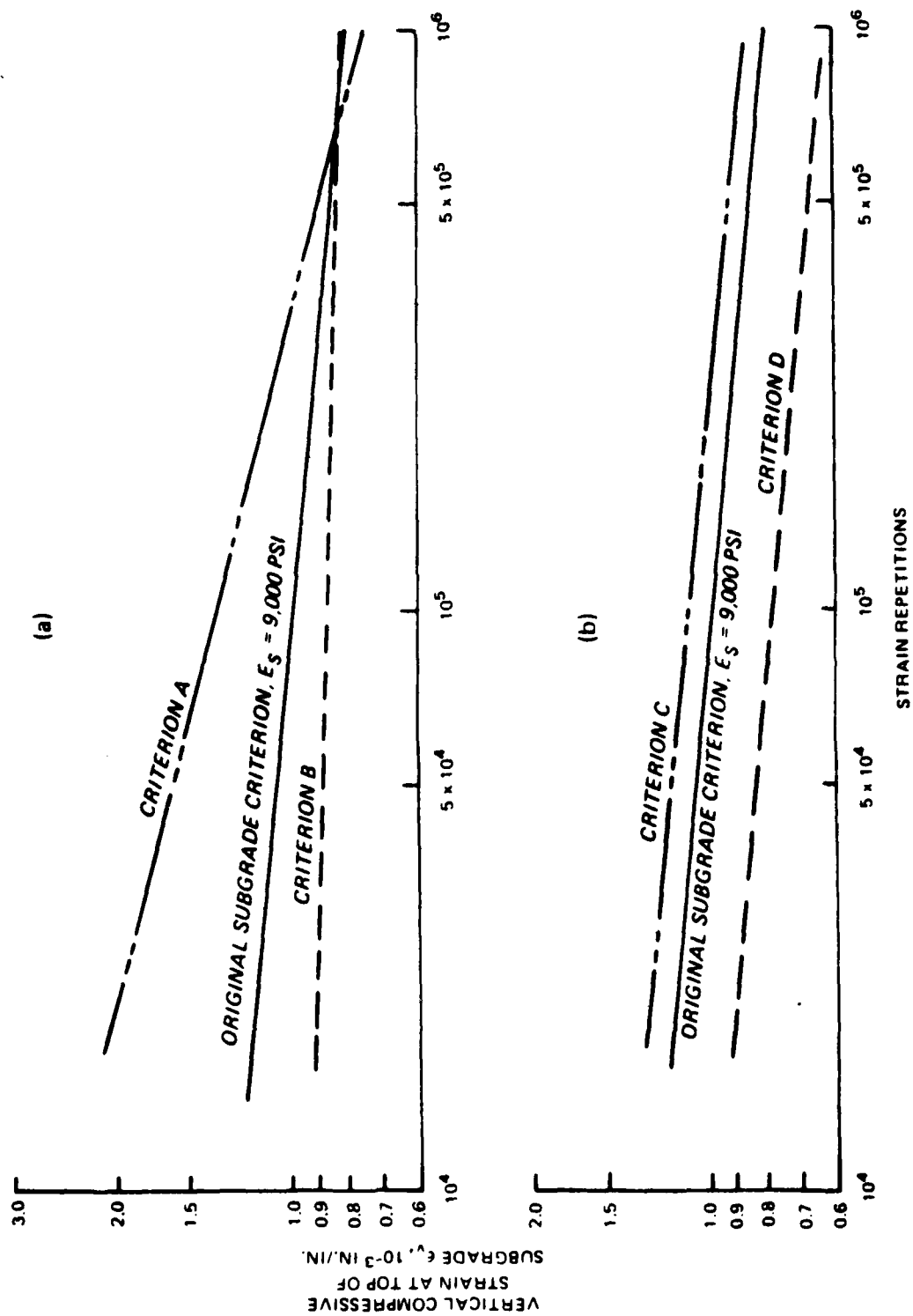


Figure 14. Subgrade strain criteria for flexible airfield pavements

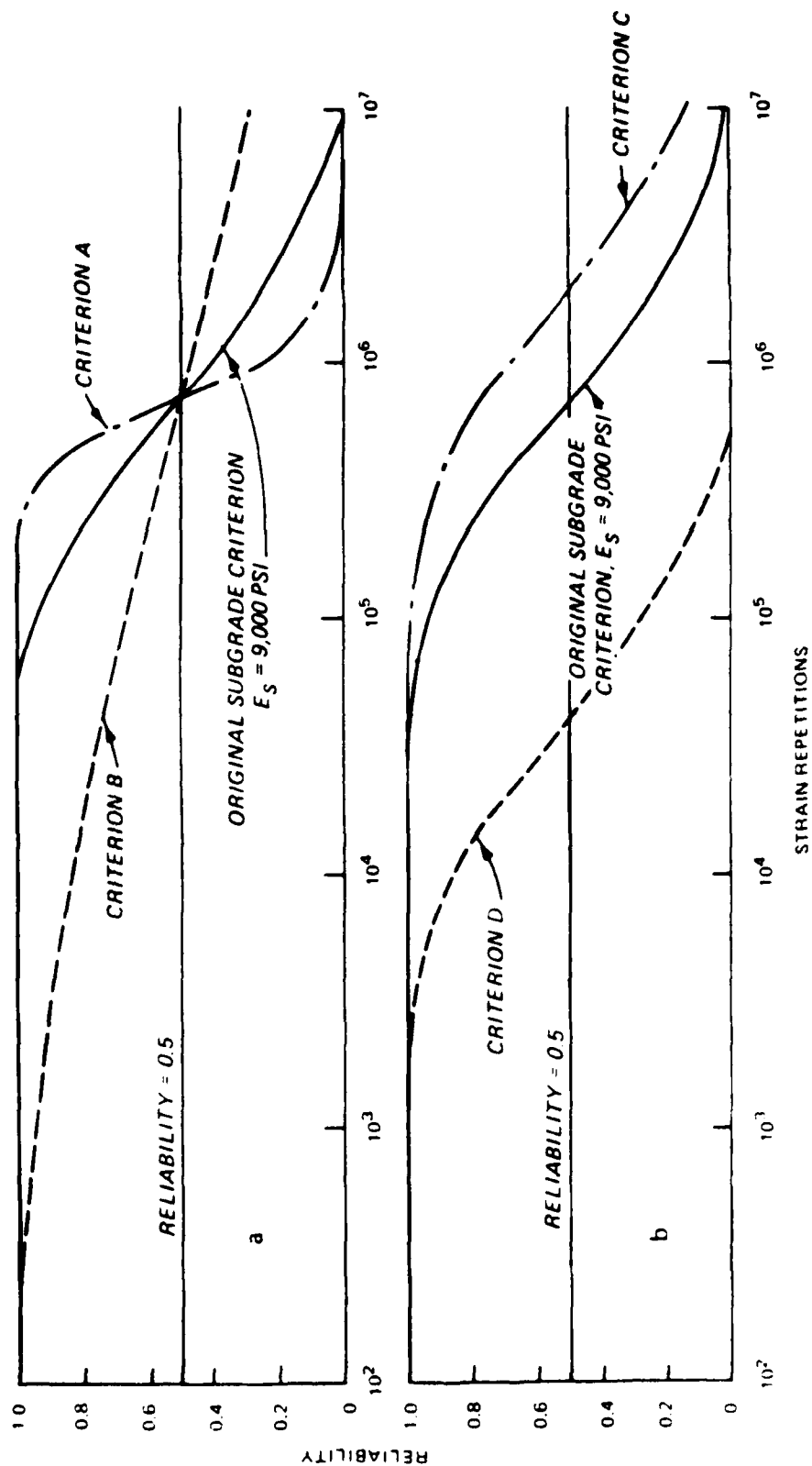


Figure 15. Relationships between reliability and strain repetitions for different subgrade failure criteria

the original strain criterion (i.e., less conservative with lower predicted strain repetition for a given strain value), and the reliability-strain repetition curve in Figure 15 (criterion D) is moved to locations of lower strain repetitions than the curve computed from the original strain criterion.

69. Strains are computed independent of the failure criterion employed. The allowable strain repetitions are estimated from strain values through the failure criterion employed. For a given range of strain values, the range of predicted strain repetitions is smaller for failure strain criterion having steeper slope (Figure 14), and thus resulting in a steeper slope of the reliability-strain repetition curve (Figure 15) and lesser uncertainty in the designed pavement. When the failure strain criterion is moved in the parallel direction (Figure 14), the resulting reliability-strain repetition curve also moves parallel in the direction of strain-repetition axis.

70. All the reliability-strain repetition curves shown in the figures have steeper slopes for the bituminous concrete strain criterion than for the subgrade strain criterion. It seems that flexible pavements designed using the Corps of Engineer's failure criteria (Figures 1 and 2) will have a greater degree of uncertainty in preventing subgrade failure than preventing fatigue cracking of bituminous concrete surface course. However, this may not be true in real cases. It is extremely important to point out at this stage that the subgrade failure criteria presented in Figure 2 are primarily based on the full-scale accelerated traffic test data while the bituminous concrete strain criteria are derived based on laboratory fatigue data on the breaking stress and strain of bituminous base-course materials (with about 5 percent air voids). Since failure criteria derived from laboratory tests do not consider the uncertainties existing in the laboratory-to-field correlations, the actual performance of the pavement will be more uncertain than is considered in the design. Even though the slopes of the reliability-strain repetition curves are steeper in the bituminous concrete strain criterion than in the subgrade strain criterion, it is not necessarily true that pavements designed using the Corps of Engineer's failure criteria will have lesser degree of uncertainty in preventing fatigue cracking of bituminous concrete surface course than preventing subgrade failure. There is a need to incorporate the field uncertainties into the laboratory determined failure criteria.

## PART VIII: CONCLUSIONS AND RECOMMENDATIONS

### Conclusions

71. Based on the analysis of RELIBISA computer program which is a layered elastic pavement design approach in terms of probability and reliability, the following conclusions can be drawn for flexible airfield pavement design using the layered elastic method.

#### ABC pavements

72. The relationships between reliability and strain repetition for various ABC thickness (Figures 3 and 4) can be used to design a pavement in terms of probability and reliability. For a desired reliability level of the design system, the ABC thickness can be varied to agree with the designed strain repetition, or the allowable strain repetition be modified for a given ABC thickness.

73. The change of the allowable strain repetition due to the change of ABC thickness is greater with respect to the subgrade strain criterion than with the bituminous concrete strain criterion (Figure 5).

74. The performance of an ABC pavement is sensitive to variations of the input parameters (in the descending order) ABC thickness  $h_1$ , gear load  $P$ , ABC modulus  $E_1$ , and subgrade modulus  $E_2$  for both bituminous concrete and subgrade strain criteria (Figures 7 and 8).

#### Three-layer conventional flexible pavements

75. The relationships between reliability and strain repetition (such as Figures 9 and 10) can be used to design a pavement in terms of probability and reliability. For a desired reliability level of the design system, the thickness of the bituminous concrete surface course or the thickness of the granular base can be varied to agree with the designed strain repetition, or the allowable strain repetition be modified for a given pavement structure.

76. For the subgrade strain failure criterion, the change of the allowable strain criterion due to the change of the thickness of bituminous concrete surface course is about the same as the change of the thickness of granular base course. Since the unit cost of granular layer is less than the bituminous concrete course, it is economically beneficial to increase the thickness of the granular base to prevent the pavement from subgrade failure.

The support from base course has relatively lesser significance in preventing fatigue cracking of the bituminous concrete surface course than to increase the thickness of the bituminous concrete layer itself, and the significance reduces rapidly as the thickness of the granular base continues to increase (Figure 11).

77. The performance of a flexible pavement is sensitive to variations of the input parameters (in the descending order) the gear load  $P$ , the thickness of the granular base  $h_2$ , the subgrade modulus  $E_3$ , the thickness of the bituminous concrete surface course  $h_1$ , the granular base modulus  $E_2$ , and the modulus of the bituminous concrete surface course  $E_1$  for the subgrade strain criterion, and to variations of  $P$ ,  $h_1$ ,  $E_1$ ,  $E_2$ ,  $h_2$ , and  $E_3$  for the bituminous concrete strain criterion. In the subgrade strain criterion the pavement performance is more sensitive to the variation of thickness of the granular base course  $h_2$  but least sensitive to the variation of modulus of the bituminous concrete surface course  $E_1$  while in the bituminous concrete strain criterion the pavement performance is more sensitive to the variation of the thickness of the bituminous concrete surface course  $E_1$  but least sensitive to the variation of subgrade modulus  $E_3$  (Figures 12 and 13).

#### General

78. The pavement performance is generally more sensitive to the variation of layer thickness than to that of the elastic modulus of the layer material. Actual variations of layer thickness in the field are known to be lesser than variations of material moduli.

79. All the reliability-strain repetition curves have steeper slope for the bituminous concrete strain failure criterion than for the subgrade strain failure criterion. It seems that flexible pavement designed using the Corps of Engineers' failure criteria (Figures 1 and 2) will have a greater degree of uncertainty in preventing subgrade failure than preventing fatigue cracking of the bituminous concrete surface course. However, this may not be true in real cases. The subgrade failure criteria presented in Figure 2 are based on the full-scale accelerated traffic test data, and the bituminous concrete strain criteria (Figure 1) are derived based on laboratory test data which do not consider the uncertainties existing in the laboratory-to-field correlations. The actual performance of the pavement with respect to fatigue cracking will be more uncertain than is considered in the design. Even though the slopes of



the reliability-strain repetition curves are steeper in the bituminous concrete strain criterion than in the subgrade strain criterion, it is not necessarily true that pavement designed using the Corps of Engineer's failure criteria will have lesser degree of uncertainty in preventing fatigue cracking of bituminous concrete surface course than preventing subgrade failure.

80. The relationships between reliability and strain repetition developed for ABC and flexible pavements (Figures 3, 4, 5, 9, 10, and 11) can be used to optimize the design. The thicknesses of the bituminous concrete and the base layers can be selected so that the pavement is failed in fatigue cracking and subgrade failure at nearly the same traffic level and the same reliability level. In some cases the subgrade modulus value may have to be varied to obtain an optimum design.

#### Recommendations

81. The reliability-strain repetition curves developed using the RELIBISA computer program are predicated by the failure criteria (Figures 1 and 2) employed in the computation. The subgrade strain failure criteria (Figure 2) are primarily based on the full-scale accelerated traffic test data, and the bituminous concrete strain failure criteria are derived based on laboratory fatigue data of bituminous base-course materials. Since failure criteria based on laboratory test data do not involve uncertainties existing in the laboratory-to-field correlations, the actual performance of such a pavement with respect to fatigue cracking will be more uncertain than is considered in the design. There is a strong need to incorporate the field uncertainties into the bituminous concrete failure criteria (Figure 1) which was determined based on controlled laboratory test data.

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Table 1  
Input Parameter Values Used in Equation 15

Strain term*	P,** lb	$E_1$ ,** psi	$E_2$ ,** psi	$h_1$ ,** psi
$\epsilon_{+++}$	30,000 + 3,000 = 33,000	1,000,000 + 100,000 = 1,100,000	9,000 + 900 = 9,900	20 + 2 = 22
$\epsilon_{++-}$	33,000	1,100,000	9,900	20 - 2 = 18
$\epsilon_{+-+}$	33,000	1,100,000	9,000 - 900 = 8,100	22
$\epsilon_{+--}$	33,000	1,100,000	8,100	18
$\epsilon_{-++}$	33,000	1,000,000 - 100,000 = 900,000	9,900	22
$\epsilon_{-+-}$	33,000	900,000	9,900	18
$\epsilon_{--+}$	33,000	900,000	8,100	22
$\epsilon_{---}$	33,000	900,000	8,100	18
$\epsilon_{-++}$	30,000 - 3,000 = 27,000	1,100,000	9,900	22
$\epsilon_{-+-}$	27,000	1,100,000	9,900	18
$\epsilon_{--+}$	27,000	1,100,000	8,100	22
$\epsilon_{---}$	27,000	1,100,000	8,100	18
$\epsilon_{-++}$	27,000	900,000	9,900	22
$\epsilon_{-+-}$	27,000	900,000	9,900	18
$\epsilon_{--+}$	27,000	900,000	8,100	22
$\epsilon_{---}$	27,000	900,000	8,100	18

\* Strain terms shown in Equation 15 for a two-layer system.

\*\* The mean values for input parameters P,  $E_1$ ,  $E_2$ , and  $h_1$  are assumed to be 30,000 lb, 1,000,000 psi, 9,000 psi, and 20 in., respectively. The standard deviations of P,  $E_1$ ,  $E_2$ , and  $h_1$  are assumed to be 3,000 lb, 100,000 psi, 900 psi, and 2 in., respectively.

Table 2  
Partial Computer Results of RELIBISA Computer Program

Reliability	Strain	Repetition*
0.9987 = 0.5 + 0.4987**	0.40817E-03 = $\bar{\epsilon} + 3\sigma_{\epsilon}^{\dagger}$	173890.14
0.9974	0.40189E-03	187906.51
0.9953	0.39561E-03	203300.71
0.9918	0.38933E-03	220233.39
0.9861	0.38305E-03	238886.85
0.9772 = 0.5 + 0.4772**	0.37677E-03 = $\bar{\epsilon} + 2\sigma_{\epsilon}^{\dagger}$	259468.84
0.9641	0.37049E-03	282215.83
0.9452	0.36421E-03	307398.31
0.9192	0.35794E-03	335325.81
0.8849	0.35166E-03	366354.06
0.8413 = 0.5 + 0.3413**	0.34538E-03 = $\bar{\epsilon} + \sigma_{\epsilon}$	400892.23
0.7881	0.33910E-03	439412.48
0.7257	0.33282E-03	482460.46
0.6554	0.32654E-03	530669.65
0.5793	0.32026E-03	584776.63
0.5000	0.31398E-03 = $\bar{\epsilon}^{\dagger\dagger}$	645640.77
0.4207	0.30770E-03	714266.76
0.3446	0.30142E-03	791834.63
0.2743	0.29514E-03	879733.76
0.2119	0.28886E-03	979606.13
0.1587 = 0.5 - 0.3413**	0.28258E-03 = $\bar{\epsilon} + \sigma_{\epsilon}$	1093397.03
0.1151	0.27630E-03	1223423.80
0.0808	0.27002E-03	1372454.52
0.0548	0.26374E-03	1543809.42
0.0359	0.25746E-03	1741489.42
0.0228 = 0.5 - 0.4772**	0.25118E-03 = $\bar{\epsilon} - 2\sigma_{\epsilon}$	1970334.63
0.0139	0.24490E-03	2236232.38
0.0082	0.23862E-03	2546371.47
0.0047	0.23234E-03	2909584.72
0.0026	0.22606E-03	3336775.09
0.0013 = 0.5 - 0.4987*	0.21979E-03 = $\bar{\epsilon} - 3\sigma_{\epsilon}$	3841485.34

\* Load repetitions are determined from failure criteria shown in Figures 1 and 2 based on the given strain value.

\*\* 0.3413, 0.4772, and 0.4987 are halves of the area within plus and minus one, two, and three standard deviations, respectively, under a normal distribution curve.

$\dagger \sigma_{\epsilon}$  is the standard deviation of strain and is equal to 0.000031398 in./in. in this example.

$\dagger\dagger \bar{\epsilon}$  is the strain computed based on mean input parameters.

Table 3  
Computations of Cumulative Damage for Three Conventional  
Flexible Pavement Sections

<u>Traffic Period*</u>	<u>Bituminous Concrete Modulus</u> $10^3$ psi	<u>Computed Horizontal Tensile Strain</u> $10^{-4}$ in./in.	<u>Number of Applied Strain Repetitions**</u>	<u>Number of Allowable Strain Repetitions</u>	<u>Damage Increment†</u>	<u>Cumulative Damage</u>
<u>5-in. Surface Course</u>						
1	1,500	2.73	9,009	11,463	0.78595	0.78595
2	1,270	2.85		13,911	0.64760	1.43355
3	920	3.15		19,144	0.47059	1.90413
4	570	3.50		42,562	0.21167	2.11580
5	360	3.90		90,814	0.09920	2.21500
6	220	4.30		215,393	0.04183	2.25683
7	180	4.45		309,878	0.02907	2.28590
8	180	4.45		309,878	0.02907	2.31498
9	260	4.15		163,546	0.05509	2.37006
10	540	3.52		48,266	0.18666	2.55672
11	1,000	3.06		17,795	0.50625	3.06297
12	1,400	2.77		12,693	0.70976	3.77273
<u>7-in. Surface Course</u>						
1	1,500	2.26	9,009	29,482	0.030558	0.30558
2	1,270	2.42		31,515	0.28586	0.59144
3	920	2.74		38,445	0.23434	0.82578
4	570	3.20		66,621	0.13523	0.96100
5	360	3.60		135,508	0.06648	1.02749
6	220	4.12		266,739	0.03377	1.06126
7	180	4.31		363,584	0.02478	1.08604
8	180	4.31		363,584	0.02478	1.11082
9	260	3.95		209,361	0.04303	1.15385
10	540	3.25		71,934	0.12524	1.27909
11	1,000	2.65		36,533	0.24660	1.52569
12	1,400	2.34		29,505	0.30534	1.83103
<u>9-in. Surface Course</u>						
1	1,500	1.86	9,009	78,078	0.11538	0.11538
2	1,270	2.02		77,775	0.11583	0.23122
3	920	2.35		82,843	0.10875	0.33997
4	570	2.75		142,134	0.06338	0.40335
5	360	3.33		200,104	0.04502	0.44837
6	220	3.90		350,954	0.02567	0.47404
7	180	4.06		490,186	0.02838	0.49242
8	180	4.06		490,186	0.01838	0.51080
9	260	3.72		282,596	0.03188	0.54268
10	540	2.80		151,551	0.05945	0.60212
11	1,000	2.27		79,211	0.11373	0.71586
12	1,400	1.94		75,329	0.11960	0.83545

\* Traffic period 1 consists of the 20 January in the 20-year design life; traffic period 2 consists of the 20 February, etc.

\*\* The number of applied strain repetitions is 200,000 (the number of total departures) divided by 1.85 (the factor for converting departures to coverages) divided by 12 (the number of traffic periods).

† The damage increment is the number of applied repetitions divided by the number of allowable repetitions.

Table 4  
Computations of Cumulative Damage of the Bituminous Concrete  
for Three All-Bituminous Concrete (ABC) Pavement Sections

<u>Traffic Period</u>	<u>Bituminous Concrete Modulus</u> $10^3$ psi	<u>Computed Horizontal Modulus</u> $10^{-4}$ in./in.	<u>Number of Applied Strain Repetitions</u>	<u>Number of Allowable Strain Repetitions</u>	<u>Damage Increment</u>	<u>Cumulative Damage</u>
<u>18-in. ABC Pavement</u>						
1	1,600	1.19	16,666	616,171	0.02705	0.02705
2	1,400	1.30		557,508	0.02989	0.05694
3	1,060	1.57		430,789	0.03869	0.09563
4	700	2.10		303,983	0.05483	0.15045
5	460	2.52		404,696	0.04118	0.19164
6	280	3.60		271,805	0.06132	0.25295
7	230	4.10		242,430	0.06875	0.32170
8	230	4.10		242,430	0.06875	0.39044
9	340	3.12		325,013	0.05128	0.44172
10	670	2.15		306,293	0.05441	0.49613
11	1,200	1.45		469,095	0.03553	0.53166
12	1,500	1.25		569,568	0.02926	0.56092
<u>22-in. ABC Pavement</u>						
1	1,600	0.81	16,666	4,217,056	0.00395	0.00395
2	1,400	1.00		2,069,987	0.00805	0.01200
3	1,060	1.20		1,651,418	0.01009	0.02210
4	700	1.61		1,147,669	0.01452	0.03662
5	460	1.94		1,496,656	0.01114	0.04775
6	280	2.70		1,145,385	0.01455	0.06230
7	230	3.08		1,013,333	0.01645	0.07875
8	230	3.08		1,013,333	0.01645	0.09520
9	340	2.37		1,285,085	0.01297	0.10817
10	670	1.65		1,150,563	0.01449	0.12265
11	1,200	1.10		1,866,973	0.00893	0.13158
12	1,500	0.95		2,246,338	0.00742	0.13900
<u>26-in. ABC Pavement</u>						
1	1,600	0.76	16,666	5,799,176	0.00287	0.00287
2	1,400	0.83		5,255,036	0.00317	0.00605
3	1,060	1.00		4,109,259	0.00406	0.01010
4	700	1.30		3,343,701	0.00498	0.01509
5	460	1.57		4,311,544	0.00387	0.01895
6	280	2.26		2,787,587	0.00598	0.02493
7	230	2.46		3,117,671	0.00535	0.03028
8	230	2.46		3,117,671	0.00535	0.03562
9	340	1.80		5,085,233	0.00328	0.03890
10	670	1.35		3,138,067	0.00531	0.04421
11	1,200	0.92		4,562,078	0.00365	0.04786
12	1,500	0.80		5,304,486	0.00314	0.05100



Table 5  
Computations of Cumulative Damage of the Subgrade for Four  
All-Bituminous Concrete (ABC) Pavement Sections

<u>Monthly Period</u>	<u>Bituminous Concrete Modulus 10<sup>3</sup> psi</u>	<u>Computed Horizontal Tensile Strain 10<sup>-4</sup> in./in.</u>	<u>Number of Applied Strain Repetitions</u>	<u>Number of Allowable Strain Repetitions</u>	<u>Damage Increment</u>	<u>Cumulative Damage</u>
<u>18-in. ABC Pavement</u>						
May	320	0.90	833	20,000	0.04	0.04
Jun	180	1.20		630	1.32	1.36
Jul	160	1.35		200	4.16	5.52
Aug	160	1.35		200	4.16	9.68
Sep	230	1.10		1,700	0.49	10.17
Oct	400	0.79		90,000	0.01	10.18
<u>21-in. ABC Pavement</u>						
Jun	180	1.02	833	4,000	0.21	0.21
Jul	160	1.10	833	1,650	0.50	0.71
Aug	160	1.10	833	1,650	0.50	1.21
Sep	230	0.89	833	22,000	0.04	1.25
<u>22-in. ABC Pavement</u>						
Jun	180	0.94	833	11,500	0.07	0.07
Jul	160	1.02	833	4,000	0.21	0.28
Aug	160	1.02	833	4,000	0.21	0.49
Sep	230	0.82	833	60,000	0.01	0.50
<u>24-in. ABC Pavement</u>						
Jun	180	0.86	833	34,000	0.02	0.02
Jul	160	0.92	833	15,000	0.05	0.07
Aug	160	0.92	833	15,000	0.05	0.12

Note: Damage was accumulated only for those months during which the computed subgrade strain was greater than or equal to  $0.78 \times 10^{-3}$  in./in.

Table 6

Performance Variations as Functions of Variations of Input Parameters  
(Subgrade Strain Criterion) All-Bituminous Concrete Pavement

Coefficient of Variation				Standard Deviation of Strain	Strain Repetitions for	
P	E <sub>1</sub>	E <sub>2</sub>	h <sub>1</sub>	$\epsilon^*, \sigma_\epsilon$	$\epsilon + \sigma_\epsilon$	$\epsilon - \sigma_\epsilon$
0.1	0.0	0.0	0.0	0.000082	254,639	2,171,960
0.0	0.1	0.0	0.0	0.000045	396,666	1,293,988
0.0	0.0	0.1	0.0	0.000037	439,129	1,156,346
0.0	0.0	0.0	0.1	0.000101	202,718	2,889,570

\* The subgrade strain computed for the pavement is 0.00082 in./in.

Table 7

Performance Variations as Functions of Variations of Input Parameters  
(Subgrade Strain Criterion) Flexible Pavement

Coefficient of Variation						Standard Deviation of Strain	Strain Repetitions for	
P	E <sub>1</sub>	E <sub>2</sub>	E <sub>3</sub>	h <sub>1</sub>	h <sub>2</sub>	$\epsilon^*, \sigma_\epsilon$	$\epsilon + \sigma_\epsilon$	$\epsilon - \sigma_\epsilon$
0.1	0.0	0.0	0.0	0.0	0.0	0.0000940	56,160	479,000
0.0	0.1	0.0	0.0	0.0	0.0	0.0000077	142,500	169,670
0.0	0.0	0.1	0.0	0.0	0.0	0.0000249	117,540	207,120
0.0	0.0	0.0	0.1	0.0	0.0	0.0000618	78,790	321,140
0.0	0.0	0.0	0.0	0.1	0.0	0.0000330	107,580	227,570
0.0	0.0	0.0	0.0	0.0	0.1	0.0000724	70,360	365,940

\* The subgrade strain computed for the pavement is 0.0009494 in./in.

Table 8  
Thickness Measurement Variations\*

<u>Year</u>	<u>Material</u>	<u>Mean Deviation from Planned Thickness, ft</u>	<u>Standard Deviation</u>	<u>Number of Measurements</u>
1962	Asphalt concrete	+0.02	0.03	823
	Cement-treated base	+0.02	0.06	934
	Aggregate base	+0.00	0.07	1,149
	Aggregate subbase	0.00	0.08	1,037
1963	Asphalt concrete	+0.01	0.03	1,327
	Cement-treated base	+0.02	0.06	1,173
	Aggregate base	0.00	0.06	1,310
	Aggregate subbase	0.00	0.09	1,183
1964- 1965	Asphalt concrete	+0.02	0.03	1,760
	Cement-treated base	+0.02	0.05	2,187
	Aggregate base	0.00	0.06	1,285
	Aggregate subbase	+0.02	0.10	1,922
1966	Asphalt concrete	+0.02	0.04	1,569
	Cement-treated base	0.00	0.06	1,569
	Aggregate base	0.00	0.07	1,272
	Aggregate subbase	+0.03	0.12	1,833
1967	Asphalt concrete	+0.01	0.03	1,838
	Cement-treated base	0.00	0.06	1,412
	Aggregate base	+0.01	0.07	1,134
	Aggregate subbase	+0.03	0.11	1,877
1968	Asphalt concrete	+0.02	0.04	1,135
	Cement-treated base	+0.01	0.05	1,156
	Aggregate base	+0.01	0.06	828
	Aggregate subbase	+0.01	0.10	1,526
1969	Asphalt concrete	+0.02	0.04	1,323
	Cement-treated base	+0.01	0.06	1,318
	Aggregate base	+0.02	0.07	1,075
	Aggregate subbase	+0.02	0.11	1,370

\* From Sherman (1971).

# APPENDIX A: EXPECTATION AND VARIANCE OF A RANDOM VARIABLE\*

1. The expectation of a discrete random variable  $x$ , denoted by  $E(x)$  or simply  $\mu_x$ , is defined as

$$E(x) = \sum_{\text{all } x_1} x_1 f(x_1) \quad (A1)$$

where

$x_1$  = all possible values of the random variable  $x$   
 $f(x_1)$  = probability-distribution (or probability-density) function which assigns the corresponding probability to each  $x_1$

2. The variance of  $x$ , denoted by  $V(x)$  or  $\sigma_x^2$ , is defined as

$$V(x) = E[(x - \mu_x)^2] = \sum_{\text{all } x_1} (x_1 - \mu_x)^2 f(x_1) \quad (A2)$$

The expectation and variance of  $x$  can also be defined as the first and second moments of  $x$  as explained below.

3. If  $x_1, x_2, \dots, x_N$  are  $N$  values of a random variable  $x$ , the  $k^{\text{th}}$  moment of  $x$  about the origin, is defined as

$$E(x^k) = \frac{x_1^k + x_2^k + \dots + x_N^k}{N} \quad (A3)$$

The first ( $k = 1$ ) moment,  $E(x^1)$ , is the expected value of  $x$ .

4. If  $x_1, x_2, \dots, x_M$  occur with frequencies of  $f_1, f_2, \dots, f_m$ , respectively,

$$E(x^k) = \bar{x}_k = \frac{f_1 x_1^k + f_2 x_2^k + \dots + f_m x_m^k}{N} \quad (A4)$$

---

\* Readers may gain further information from Harr (1977). References cited in this appendix are included in the References at the end of the main text.

where  $N = \sum_{i=1}^M f_i$ .

5. The  $k^{\text{th}}$  moment about the mean  $\bar{x}_1$  or the  $k^{\text{th}}$  central moment of a random variable  $x$  is defined as

$$E (x - \bar{x}_1)^k = \sum_{i=1}^N \frac{x_i - \bar{x}_1^k}{N} \quad (\text{A5})$$

The second central moment,  $k = 2$ , is the variance of  $x$ ,  $V(x)$ . For

grouped data with frequencies  $f_1, f_2, \dots, f_M$ ,  $N = \sum_{i=1}^M f_i$ ,

$$E \left[ (x - \bar{x}_1)^k \right] = \frac{f_1 (x_1 - \bar{x}_1)^k + f_2 (x_2 - \bar{x}_1)^k + \dots + f_M (x_M - \bar{x}_1)^k}{N} \quad (\text{A6})$$

## APPENDIX B: INPUT GUIDE AND RUN EXAMPLE OF THE RELIBISA COMPUTER PROGRAM

The input guide for the program is given in Table B1 with detailed explanations of each entry presented below:

a. Item 1: Number of Runs Card

<u>Notes</u>	<u>Variables</u>	<u>Entry</u>
	NRUN	Number of runs to be computed

b. Item 2: Search for Locations of Maximum Strains Card

<u>Notes</u>	<u>Variables</u>	<u>Entry</u>
(1)	ONLY	EQ. 1 Search for locations of maximum strains in each term in Equations 12 to 16.  EQ. 0 Determine the locations of the maximum strains only for the case of mean input parameters.

**Notes:**

(1) The locations of the maximum strains at the bottom of the bituminous concrete surface course and at the subgrade surface are first determined with the mean values of the input parameters. To save computer time, the locations are arbitrarily determined either at the centroid of the gear assembly or directly beneath one wheel load. The same locations can be used to compute the strains in each term of Equations 12 to 16 (of the main text) to save computer time (i.e., ONLY = 0), or the locations of the maximum strains are searched separately in each term of Equations 12 to 16 (i.e., ONLY = 1).

c. Item 3: Aircraft Type and Number of Layer Card

<u>Notes</u>	<u>Variables</u>	<u>Entry</u>
(1)	AIPPLN	Gear assembly category
(2)	NLAYS	Number of layers of the flexible pavement
(3)	ISMO	EQ. 0 rough computational procedure EQ. 1 smooth computational procedure
(4)	IREDD	EQ. 0 AK(I) is input in ITEM 4 EQ. 1 ALK(I) is input in ITEM 4

**Notes:**

(1) The gear assembly category of the design aircraft is shown in Figure B1.

(2) NLAYS cannot be greater than three and smaller than two in RELIBISA program.

(3) Smooth computation involves more computation time and thus have more accurate results. It is suggested that ISMO = 1 be used.

(4) It is suggested that IRED = 1 be used.

d. Item 4: Layer Information Card

<u>Notes</u>	<u>Variables</u>	<u>Entry</u>
	E(1)	Elastic modulus of the material in first layer
	NU(1)	Poisson's ratio of the material in first layer
	Thick(1)	Thickness of the first layer
(1)	AK(1)	Reduced interface compliance of the first layer
	.	
	.	
	E(NLAYS-1)	
	NU(NLAYS-1)	
	Thick (NLAYS-1)	
	AK(NLAYS-1)	

**Notes:**

- (1) For complete adhesion between layers 1 and i+1 set  $AK(i) = ALK(i) = 0$   
 For complete frictionless slip between layers set  $(E_1/1 + N_1) AK(i) = ALK(i) \geq 1,000$ . It is suggested that a value of 1,000,000 be used.

e. Item 5: Subgrade Card

<u>Notes</u>	<u>Variables</u>	<u>Entry</u>
	E(NLAYS)	Modulus of the subgrade
	NU(NLAYS)	Poisson's ratio of the subgrade

f. Item 6: Aircraft Information Card

<u>Notes</u>	<u>Variables</u>	<u>Entry</u>
(1)	LDSTR	The wheel load
	RADIU	Radius of the wheel load
(2)	A	Gear dimension A
(2)	B	Gear dimension B
(2)	C	Gear dimension C
(2)	D	Gear dimension D

**Notes:**

- (1) All the wheel loads in the gear assembly should have the same loaded area and load magnitude.
- (2) The gear dimensions are shown in Figure B1.

**g. Item 7: Reliability and Variability Card**

<u>Notes</u>	<u>Variables</u>	<u>Entry</u>
(1)	RELIBITY	Inherent reliability level of the pavement design system to be analyzed
(2)	CVP	Coefficient of variation of the wheel load P
(2)	CVE(1)	Coefficient of variation of the modulus of the surface layer
(2)	CVE(2)	Coefficient of variation of the modulus of the second layer
(2)	CVE(3)	Coefficient of variation of the modulus of the third layer
(2)	CVH(1)	Coefficient of variation of the thickness of the first layer
(2)	CVH(2)	Coefficient of variation of the thickness of the second layer

**Notes:**

- (1) For Corps of Engineers flexible pavement design system, RELIBITY = 0.5 is suggested.
- (2) The coefficient of variation is defined to the ratio of the standard deviation to the mean. For a two-layer system,  $CVE(3) = CVH(2) = 0$ , as E(2) is the modulus of the subgrade.



Table B1

Input Guide for RELIBISA -- Layered Elastic Analysis of Flexible  
Pavements in Probabilistic and Reliability Terms

---

ITEM 1. Number of Runs Card

NRUN

ITEM 2. Search for Location of Maximum Strains Card

ONLY

ITEM 3. Aircraft Type and Number of Layer Card

AIPPLN, NLAYS, ISMO, IRED

ITEM 4. Layer Information Card

E(I), NU(I), Thick(I), AK(I), I=1, (NLAYS - 1)

ITEM 5. Subgrade Card

E(NLAYS), NU(NLAYS)

ITEM 6. Aircraft Information Card

LDSTR, RADIU, ADIM, BDIM, CMIM, DDIM

ITEM 7. Reliability and Variability Card

RELIBITY, CVP, CVE(1), CVE(2), CVE(3), CVH(1), CVH(2)

**AIRPLANE  
(AIPAN)  
DESIGNATIONS**

**GEAR CONFIGURATION**

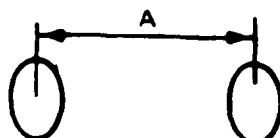
**A   B   C   D**

1



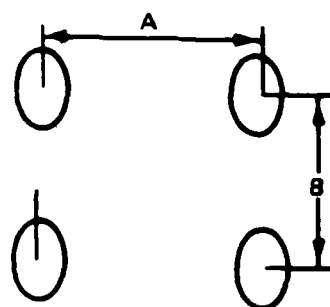
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2



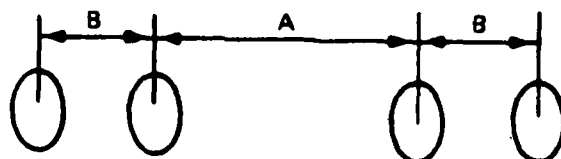
A   o   o   o

3



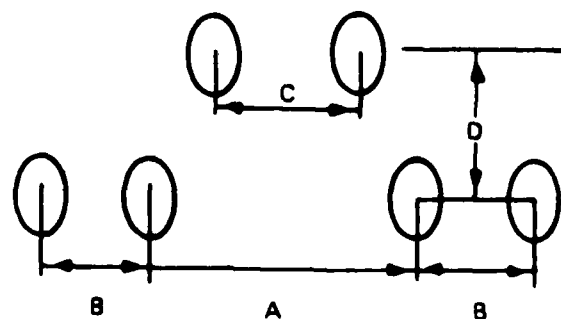
A   B   o   o

4



A   B   o   o

5



A   B   C   D

Figure B1. Dimensions of aircraft gear configurations

END

DATE

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FEB.

1988